

حلقہ سمت: گولڈن ← تعبیر هندسی ← تعبیر فیزیکی ← کرل ← تعبیر فیزیکی

$$[\hat{A} \cdot (\hat{B} \times \hat{C})]^2 = 1 - \cos^2 \alpha - \omega^2 [\cos^2 \beta + \cos^2 \gamma - 2 \cos \alpha \cos \beta \cos \gamma]$$

حجم متوازی السطوح با بزرگ α, β, γ

تعلق با بزرگ: $\omega = 1 \Rightarrow \omega = \pm 1$

حالت خاص: $\hat{x} \times (\hat{x} \times \hat{y}) = \hat{A} \times (\hat{B} \times \hat{C})$

$$\hat{x} \times (\hat{x} \times \hat{y}) = \hat{x} \times \hat{z}$$

$\omega = 1 \Rightarrow *$

ناتمام: $\vec{A} \times (\vec{B} \times \vec{C}) = \omega [\vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B})]$

$$= \omega [\hat{x} (\hat{x} \cdot \hat{y}) - \hat{y} (\hat{x} \cdot \hat{x})] = -\omega \hat{y} *$$

گولڈن

مثال: $\phi(x, y, z) = x^2 + y^2 + z^2$

$\phi(x, y, z)$: کثرت تابع (کثرت)

$$\left. \begin{matrix} x \rightarrow x' \\ y \rightarrow y' \\ z \rightarrow z' \\ \phi \rightarrow \phi' \end{matrix} \right\} \rightarrow \phi'(x', y', z') = \phi(x, y, z) \quad (1)$$

تغییرات تابع ϕ در مرتبه اول (تقریباً) $\phi(x, y, z)$ به نقطه $(x+dx, y+dy, z+dz)$ برسی کنیم

$$d\phi = \phi(x+dx, y+dy, z+dz) - \phi(x, y, z)$$

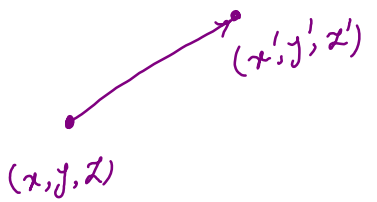
$$d\phi = \left[\phi(x+dx, y, z) - \phi(x, y, z) \right] +$$

$$\left[\phi(x, y+dy, z) - \phi(x, y, z) \right] + \left[\phi(x, y, z+dz) - \phi(x, y, z) \right]$$

تقریباً: $f(x+\epsilon) = f(x) + \epsilon \frac{\partial f}{\partial x} + \frac{1}{2!} \epsilon^2 \frac{\partial^2 f}{\partial x^2} + \dots$

$$\Rightarrow f(x+\epsilon) - f(x) = \epsilon \frac{\partial f}{\partial x}$$

$$\Rightarrow d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz$$



$$\textcircled{1} : \frac{\partial \phi(x'_1, x'_2, x'_3)}{\partial x'_i} = \frac{\partial \phi(x_1, x_2, x_3)}{\partial x_j} = \sum_j \frac{\partial \phi}{\partial x_j} \frac{\partial x_j}{\partial x'_i} = \sum_j a_{ij} \frac{\partial \phi}{\partial x_j}$$

$$\rightarrow \frac{\partial \phi'}{\partial x'_i} = \sum_j a_{ij} \frac{\partial \phi}{\partial x_j}$$

$$\text{مثال ۱: } \vec{A}'_i = \sum_j a_{ij} A_j$$

نقطه‌های یک بردار هستند

$$\frac{\partial \phi}{\partial x_i}$$

$\vec{\nabla} \phi$

$$\rightarrow \vec{\nabla} \phi = \frac{\partial \phi}{\partial x} \hat{x} + \frac{\partial \phi}{\partial y} \hat{y} + \frac{\partial \phi}{\partial z} \hat{z} = \sum_i \frac{\partial \phi}{\partial x_i} \hat{x}_i$$

$$\Rightarrow \vec{\nabla} = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z}$$

انرژی پتانسیل

$$dU = -\vec{F} \cdot d\vec{r} = \vec{\nabla} U \cdot d\vec{r} \Rightarrow dU = \vec{\nabla} U \cdot d\vec{r}$$

$$dU = -\int_1^2 \vec{F} \cdot d\vec{r} \Rightarrow \vec{F} = -\vec{\nabla} U$$

مثال ۱

$$dU = -F dx \rightarrow F = -dU/dx \text{ (حالت یک بعدی)}$$

$$dU = -F dx$$

مثال ۲: اگر پتانسیل یک سیستم فقط به اندازه فاصله یک نقطه تا مبدأ مختصات بستگی داشته باشد

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

کولن

$$V(r) = G \frac{m_1 m_2}{r}$$

گرانش

نکته: $V = V(r)$ ، بنابراین V را نسبت آریید.

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\vec{\nabla} V = \frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} = \frac{dV}{dr} \left(\frac{x}{r} \hat{x} + \frac{y}{r} \hat{y} + \frac{z}{r} \hat{z} \right)$$

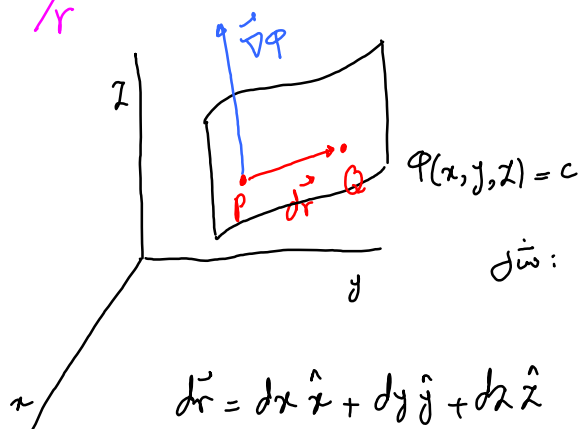
$$= \frac{1}{r} \frac{dV}{dr} (x\hat{x} + y\hat{y} + z\hat{z})$$

$$\Rightarrow \vec{\nabla} V = \frac{dV}{dr} \left(\frac{\vec{r}}{r} \right) = \frac{dV}{dr} \hat{r}$$

$$\frac{\partial r}{\partial x} = \frac{1}{2} x 2x (x^2 + y^2 + z^2)^{-1/2}$$

$\frac{dV}{dr} \left(\frac{\partial r}{\partial x} \right) x/r$

$$\rightarrow \partial r / \partial x = x/r$$



$$\text{مثال: } \phi(x, y, z) = x^2 + y^2 + z^2 = R^2$$

$$d\vec{r} = dx \hat{x} + dy \hat{y} + dz \hat{z}, \quad \vec{\nabla} \phi = \frac{\partial \phi}{\partial x} \hat{x} + \frac{\partial \phi}{\partial y} \hat{y} + \frac{\partial \phi}{\partial z} \hat{z}$$

تکبیر هندسی گرادیان

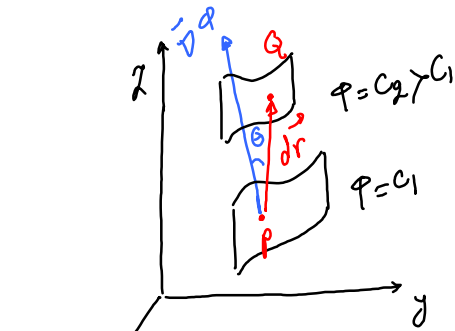
$$\vec{\nabla} \phi \cdot d\vec{r} = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy + \frac{\partial \phi}{\partial z} dz = d\phi$$

حبره روی سطح ثابت ϕ هست، $d\phi = 0$

$$\Rightarrow \vec{\nabla} \phi \cdot d\vec{r} = 0 \Rightarrow \vec{\nabla} \phi \perp d\vec{r} \text{ عمود است.} \rightarrow \vec{\nabla} \phi \text{ به سطح ثابت } \phi \text{ عمود است.}$$

* تمرین ۱: برقرار زمال سطح $x^2 + 2xz = 4$ را در نقطه $(2, -2, 3)$ بیابید.

* تمرین ۲: زاویه بین دو سطح $x^2 + y^2 + z^2 = 9$ و $z = x^2 + y^2 - 3$ را در نقطه $(2, -1, 2)$ بیابید.

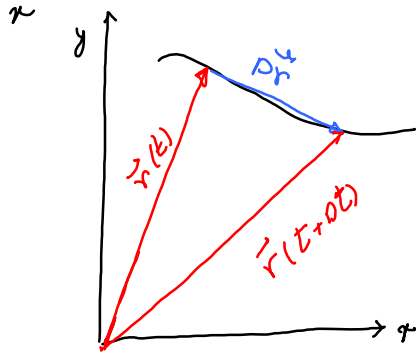


$$\text{پاسخ: } \vec{\nabla} \phi \cdot d\vec{r} = d\phi = c_2 - c_1$$

$$\Rightarrow |\vec{\nabla} \phi| |d\vec{r}| \cos \theta = c_2 - c_1$$

و θ زاویه ماکزیمم است. $\theta = 0$ باشد یعنی جابجایی در جهت $\vec{\nabla} \phi$ باشد

چون $c_2 > c_1$ است پس $\vec{\nabla} \phi$ در جهت افزایش ϕ است.



$$\lim_{\Delta t \rightarrow 0} \frac{d\vec{r}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{x} + \frac{dy}{dt} \hat{y} + \frac{dz}{dt} \hat{z}$$

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \quad \text{مبنای استاندارد}$$

$$\Rightarrow \vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \quad \text{نمونه یک اسکالر است.}$$

مثال:

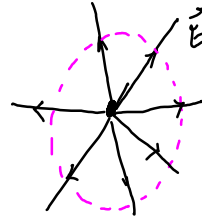
$$\vec{\nabla} \cdot (\vec{r} f(r)) = \frac{\partial}{\partial x} [x f(r)] + \frac{\partial}{\partial y} [y f(r)] + \dots$$

$$\frac{\partial}{\partial x} [x f(r)] = f(r) + x \frac{\partial f(r)}{\partial x} = f(r) + x \frac{df}{dr} \frac{\partial r}{\partial x} = f(r) + x \frac{1}{r} \frac{df}{dr}$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{r} f(r)) = 3f(r) + \frac{1}{r} \frac{df}{dr} [x^2 + y^2 + z^2] = 3f(r) + r \frac{df}{dr}$$

$$\vec{A} \cdot \vec{\nabla} = A_x \frac{\partial}{\partial x} + A_y \frac{\partial}{\partial y} + A_z \frac{\partial}{\partial z} \rightarrow (\vec{A} \cdot \vec{\nabla}) \phi = A_x \frac{\partial \phi}{\partial x} + \dots + \dots$$

الكهرباء مغناطيسية : $\vec{\nabla} \cdot \vec{B} = 0$ $\rightarrow$ source, sink
مركبة مغناطيسية مغناطيسية



$\vec{\nabla} \times$ كرد

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z}$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \hat{x} \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) + \hat{y} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) + \dots$$

$$\vec{\nabla} \times (f \vec{v}) = (\vec{\nabla} f) \times \vec{v} + f \vec{\nabla} \times \vec{v}$$

$$\vec{\nabla} \times (f \vec{r}) = (\vec{\nabla} f) \times \vec{r} + f \vec{\nabla} \times \vec{r}$$

طالع : $f = f(r) \rightarrow$ $\vec{\nabla} f(r) = \frac{df}{dr} \hat{r} \Rightarrow \vec{\nabla} \times (f \vec{r}) = \left(\frac{df}{dr} \hat{r} \right) \times \vec{r} = 0$

$$\vec{L} = \vec{r} \times \vec{p} \rightarrow \vec{L} = -i\hbar \vec{r} \times \vec{\nabla} : Q.M.$$

نلاحظ : $\vec{A} \times \vec{A} = 0$ لأن $\vec{A} \times \vec{A}$ دائمًا صفرية.

$$\vec{p} = -i\hbar \vec{\nabla} : Q.M.$$

$$\vec{L} \times \vec{L} = i\hbar \vec{L} \quad * \text{مترين : ناك دهي}$$