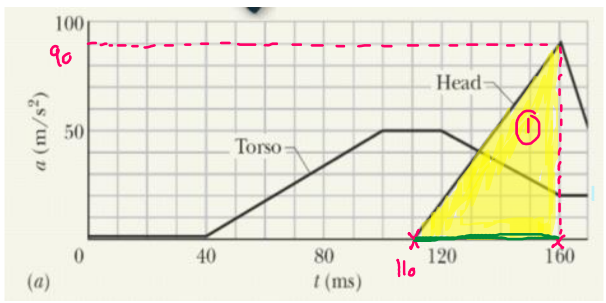
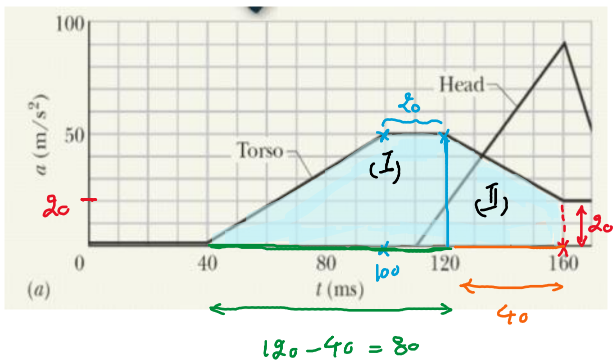
65 E BIO CALC FCP Figure 2.6.2a gives the acceleration of a volunteer's head and torso during a rear-end collision. At maximum head acceleration, what is the speed of (a) the head and (b) the torso?



$$a = \frac{dv}{dt} \rightarrow v = \int a\,dt$$

a) $S_1 = \frac{\overbrace{(160-110)}^{50} \times 10^{-3} \times 90}{2} = 2.25$

$\Rightarrow$ speed of the head $= 2.25$ m/s



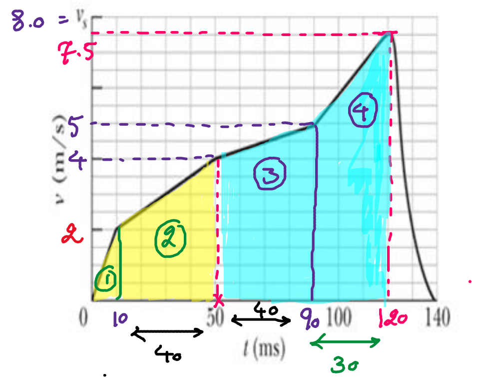
$$S_I = \frac{\overbrace{(20+80)}^{100} \times 10^{-3} \times 50}{2} = 2.5$$

$$S_{II} = \frac{\overbrace{(20+50)}^{70} \times \cancel{40}^{20} \times 10^{-3}}{\cancel{2}} = 1.4$$

$\Rightarrow$ speed of torso $= 2.5 + 1.4 = 3.9$ m/s

66 M BIO CALC FCP In a forward punch in karate, the fist begins at rest at the waist and is brought rapidly forward until the arm is fully extended. The speed $v(t)$ of the fist is given in Fig. 2.22 for someone skilled in karate. The vertical scaling is set by $v_s = 8.0$ m/s. How far has the fist moved at (a) time $t = 50$ ms and (b) when the speed of the fist is maximum?

$$v = \frac{dx}{dt} \rightarrow x = \int v\,dt$$



a)

$$S_1 = \frac{10 \times 10^{-3} \times 2}{2} = 0.01$$

$$S_2 = \frac{(2+4) \times \cancel{40}^{20} \times 10^{-3}}{\cancel{2}} = 0.12$$

$\rightarrow S_1 + S_2 = 0.01 + 0.12 = 0.13 \rightarrow d(t=0, t=50) = 0.13$ m

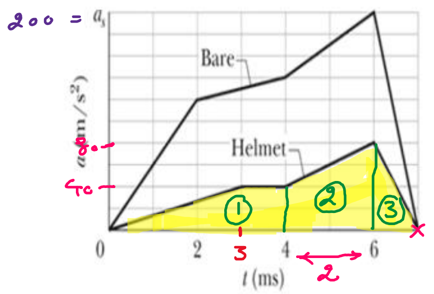
b) $S_3 = \frac{(4+5) \times \cancel{40}^{20} \times 10^{-3}}{\cancel{2}} = 0.18$

$$S_4 = \frac{(4+7.5) \times 30 \times 10^{-3}}{2} = 0.1875$$

$\rightarrow S_1 + S_2 + S_3 + S_4 = 0.01 + 0.12 + 0.18 + 0.1875 = 0.4975 \rightarrow d(t=0, t=120) = 0.50$ m

67 M BIO CALC When a soccer ball is kicked toward a player and the player deflects the ball by "heading" it, the acceleration of the head during the collision can be significant. Figure 2.23 gives the measured acceleration $a(t)$ of a soccer player's head for a bare head and a helmeted head, starting from rest. The scaling on the vertical axis is set by $a_s = 200\ \text{m/s}^2$. At time $t = 7.0$ ms, what is the difference in the speed acquired by the bare head and the speed acquired by the helmeted head?

$$a = \frac{dv}{dt} \rightarrow v = \int a\,dt$$



with helmet:

$$S_1 = \frac{(4+1)\times 10^{-3} \times \cancel{40}^{20}}{\cancel{2}} = 0.1$$

$$S_2 = \frac{(40+80)\times \cancel{2} \times 10^{-3}}{\cancel{2}} = 0.12$$

$$S_3 = \frac{\cancel{80}^{40} \times 1 \times 10^{-3}}{\cancel{2}} = 0.04$$

$$\Rightarrow S_1 + S_2 + S_3 = 0.26 \Rightarrow v_{helmeted} = 0.26\ \text{m/s}$$

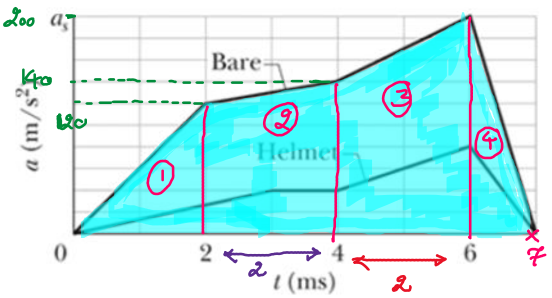
bare

$$S_1 = \frac{120 \times \cancel{2} \times 10^{-3}}{\cancel{2}} = 0.12$$

$$S_2 = \frac{(120+140)\times \cancel{2} \times 10^{-3}}{\cancel{2}} = 0.26$$

$$S_3 = \frac{(140+200)\times \cancel{2} \times 10^{-3}}{\cancel{2}} = 0.34$$

$$S_4 = \frac{\cancel{200}^{100} \times 1 \times 10^{-3}}{\cancel{2}} = 0.1$$

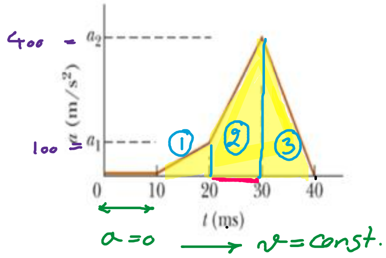


$$S_1 + S_2 + S_3 + S_4 = 0.12 + 0.26 + 0.34 + 0.1 = 0.82 \Rightarrow v_{unhelmeted} = 0.82\ \text{m/s}$$

$$\Delta v = v_{unhelmeted} - v_{helmeted} = 0.82 - 0.26 = 0.56\ \text{m/s}$$

68 M BIO CALC FCP A salamander of the genus *Hydromantes* captures prey by launching its tongue as a projectile: The skeletal part of the tongue is shot forward, unfolding the rest of the tongue, until the outer portion lands on the prey, sticking to it. Figure 2.24 shows the acceleration magnitude a versus time t for the acceleration phase of the launch in a typical situation. The indicated accelerations are $a_2 = 400\ \text{m/s}^2$ and $a_1 = 100\ \text{m/s}^2$. What is the outward speed of the tongue at the end of the acceleration phase?

$a = \frac{dv}{dt} \rightarrow v = \int a\,dt$



$S(t=40) = ?$

$S_1 = \frac{100 \times 10 \times 10^{-3}}{2} = 0.5$

$S_2 = \frac{(400+100) \times 10 \times 10^{-3}}{2} = 2.5$

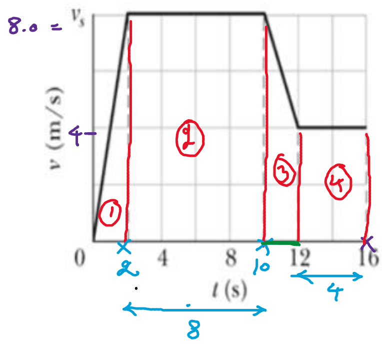
$S_3 = \frac{400 \times 10 \times 10^{-3}}{2} = 2$

$\rightarrow S_1 + S_2 + S_3 = 5.0 \rightarrow \Delta v = 5.0\ \text{m/s} \rightarrow S(t=40) = 5.0\ \text{m/s}$

$\Delta v = v(t=40) - v(t=0)$, $v(t=0) = 0$

69 M BIO CALC How far does the runner whose velocity–time graph is shown in Fig. 2.25 travel in 16 s? The figure's vertical scaling is set by $v_s = 8.0$ m/s.

$v = \frac{dx}{dt} \rightarrow x = \int v\,dt$



$S_1 = \frac{8 \times 2}{2} = 8$

$S_2 = 8 \times 8 = 64$

$S_3 = \frac{(8+4) \times 2}{2} = 12$

$S_4 = 4 \times 4 = 16$

$\rightarrow S_1 + S_2 + S_3 + S_4 = 100$

$\Downarrow$

$d(t=0, t=16) = 100\ \text{m}$

70 H CALC Two particles move along an x axis. The position of particle 1 is given by $x = 6.00t^2 + 3.00t + 2.00$ (in meters and seconds); the acceleration of particle 2 is given by $a = -8.00t$ (in meters per second squared and seconds) and, at $t = 0$, its velocity is 20 m/s. When the velocities of the particles match, what is their velocity?

particle 1 : $x = 6t^2 + 3t + 2 \longrightarrow v = \frac{dx}{dt} = 6 \times 2t + 3 \Rightarrow v_1 = 12t + 3$ *

particle 2 : $a = -8t$, $v(t=0) = 20\ \text{m/s}$

$\frac{d}{dt}\left(\frac{1}{2}t^2 + C\right) = \frac{1}{2} \times 2t + 0 = t$

$v_2 = \int a\,dt = \int (-8t)\,dt = -8\int \underbrace{t\,dt}_{\frac{1}{2}t^2 + C} = -4t^2 + C$

$v_2 = -4t^2 + C \longrightarrow v_2(t=0) = C \longrightarrow C = 20 \longrightarrow v_2 = -4t^2 + 20$ *'

$a x^2 + 2bx + c = 0$

$x = \frac{-b \mp \sqrt{b^2 - ac}}{a}$

$v_1 = v_2 \Longrightarrow 12t + 3 = -4t^2 + 20 \Longrightarrow 4t^2 + 12t - 17 = 0$

$a = 4,\ b = 6,\ c = -17$

$\rightarrow t = \frac{-6 \mp \overbrace{\sqrt{36 + 68}}^{10.198}}{4}$

$\oplus$: $t = 1.05\ \text{s} \xrightarrow{*} v = 12 \times 1.05 + 3 = 15.6\ \text{m/s}$

$\ominus$: $t = -4.05\ \text{s} \xrightarrow{*} v = 12 \times (-4.05) + 3 = 45.6\ \text{m/s}$