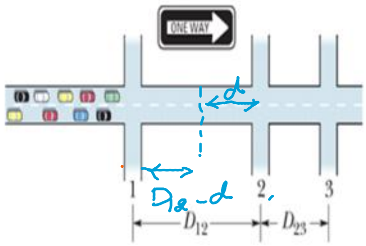
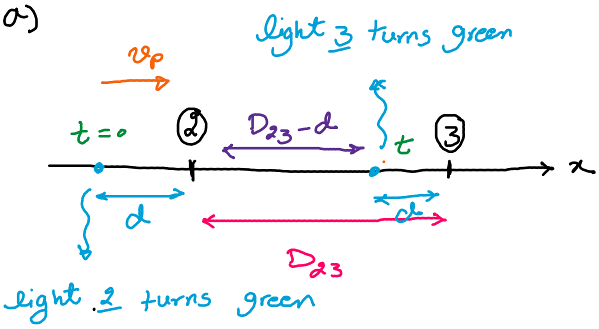
76 GO FCP Figure 2.27 shows part of a street where traffic flow is to be controlled to allow a *platoon* of cars to move smoothly along the street. Suppose that the platoon leaders have just reached intersection 2, where the green light appeared when they were distance d from the intersection. They continue to travel at a certain speed v_p (the speed limit) to reach intersection 3, where the green appears when they are distance d from it. The intersections are separated by distances D_{23} and D_{12}. (a) What should be the time delay of the onset of green at intersection 3 relative to that at intersection 2 to keep the platoon moving smoothly?

Suppose, instead, that the platoon had been stopped by a red light at intersection 1. When the green comes on there, the leaders require a certain time t_r to respond to the change and an additional time to accelerate at some rate a to the cruising speed v_p. (b) If the green at intersection 2 is to appear when the leaders are distance d from that intersection, how long after the light at intersection 1 turns green should the light at intersection 2 turn green?



a)

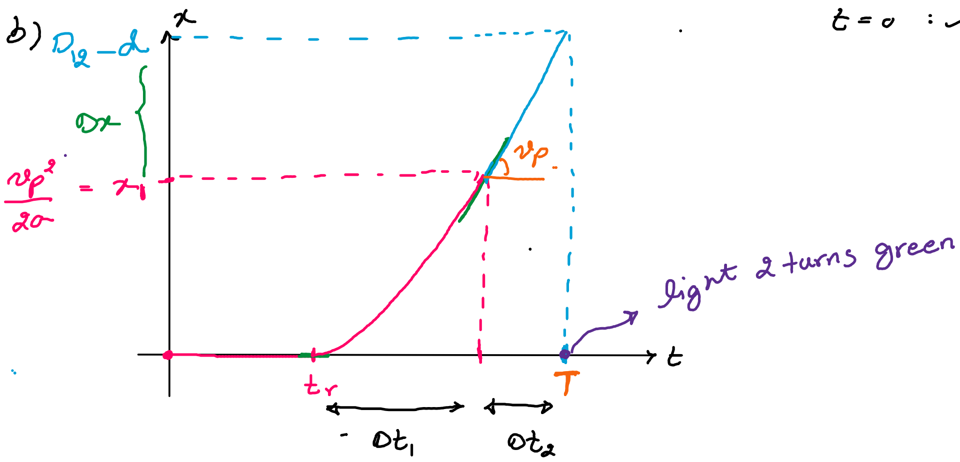


$$\rightarrow \quad t = \frac{d}{v_p} + \frac{D_{23}-d}{v_p} = \boxed{\frac{D_{23}}{v_p}}$$

$x=0$: intersection 1

$t=0$: light 1 turns green

b)



$$v^2 - \cancel{v_0}^2 = 2a(x - \cancel{x_0})$$

$$v_p^2 = 2a x_1 \rightarrow x_1 = \frac{v_p^2}{2a}$$

acceleration phase: $v - \cancel{v_0} = at \rightarrow \Delta t_1 = \frac{v_p}{a}$

(a = const.)

v = const.: $\underbrace{x - x_0}_{\Delta x} = v_p t \rightarrow \Delta t_2 = \frac{\Delta x}{v_p}$

$$\Delta x = D_{12} - d - x_1 = D_{12} - d - \frac{v_p^2}{2a}$$

$$\rightarrow \Delta t_2 = \frac{D_{12} - d - \frac{v_p^2}{2a}}{v_p} = \frac{D_{12}-d}{v_p} - \frac{v_p}{2a}$$

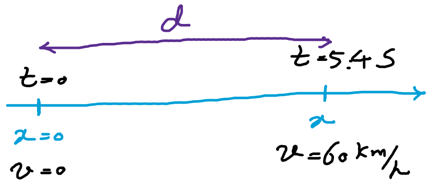
$$\rightarrow T = t_r + \Delta t_1 + \Delta t_2 = t_r + \frac{v_p}{a} + \frac{D_{12}-d}{a} - \frac{v_p}{2a} = \boxed{t_r + \frac{D_{12}-d}{a} + \frac{v_p}{2a}}$$

77 SSM A hot rod can accelerate from 0 to 60 km/h in 5.4 s. (a) What is its average acceleration, in m/s^2, during this time? (b) How far will it travel during the 5.4 s, assuming its acceleration is constant? (c) From rest, how much time would it require to go a distance of 0.25 km if its acceleration could be maintained at the value in (a)?

a) $v_1 = 0$, $v_2 = 60\ km/h$, $\Delta t = 5.4\ s$, $\bar{a} = ?\ m/s^2$

$$v_2 = 60 \times \frac{10^3\ m}{3600\ s} = 16.7\ m/s$$

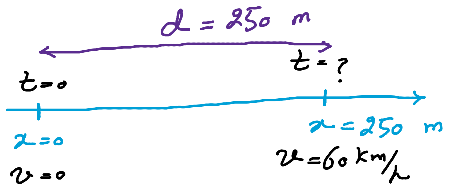
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{\Delta t} = \frac{16.7 - 0}{5.4} = 3.086 \simeq 3.1\ m/s^2$$



b) $a = Const. = \bar{a} = 3.086\ m/s^2$, $d = ?$

$$x = \frac{1}{2}at^2 + \cancel{v_0 t} + \cancel{x_0} \Longrightarrow x = \frac{1}{2} \times 3.086 \times (5.4)^2 \simeq 44.99 \simeq 45\ m$$

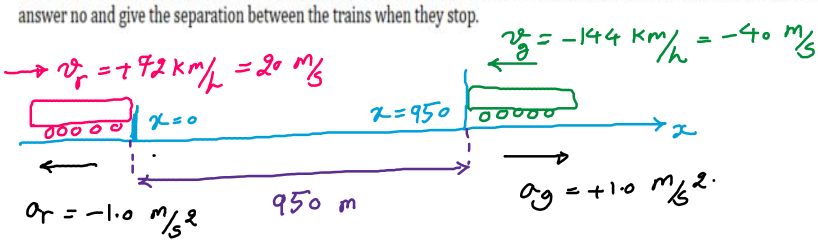
c) $a = const. = \bar{a} = 3.086\ m/s^2$, $v_0 = 0$, $d = 0.25\ km$, $t = ?$



$$x = \frac{1}{2}at^2 \longrightarrow 250 = \frac{1}{2} \times 3.086 \times t^2$$

$$\longrightarrow t = \sqrt{\frac{2 \times 250}{3.086}} \simeq 12.73 \simeq 13\ s$$

78 GO A red train traveling at 72 km/h and a green train traveling at 144 km/h are headed toward each other along a straight, level track. When they are 950 m apart, each engineer sees the other's train and applies the brakes. The brakes slow each train at the rate of $1.0\ m/s^2$. Is there a collision? If so, answer yes and give the speed of the red train and the speed of the green train at impact, respectively. If not, answer no and give the separation between the trains when they stop.



$$v_r = 72\ km/h \times \frac{10^3\ m}{3600\ s} = 20\ m/s$$

$$v_g = -144\ km/h \times \frac{10^3\ m}{3600\ s} = 40\ m/s$$

$x = \frac{1}{2}at^2 + v_0 t + x_0 \longrightarrow$ red train: $x_r = \frac{1}{2} \times (-1.0)t^2 + 20t + 0$ *

green train: $x_g = \frac{1}{2} \times (1.0)t^2 - 40t + 950$ **

$$x_r = x_g \Longrightarrow -\frac{1}{2}t^2 + 20t = \frac{1}{2}t^2 - 40t + 950 \longrightarrow t^2 - 60t + 950 = 0$$

$$ax^2 + 2bx + c = 0 \longrightarrow x = \frac{-b \mp \sqrt{b^2 - ac}}{a}$$

$$a = 1 ,\ b = -30 ,\ c = 950 \longrightarrow t = \frac{30 \mp \sqrt{900 - 950 = -50}}{1} \rightsquigarrow \text{No collison (?)}$$

$$v = \frac{dx}{dt} \longrightarrow v_r = \frac{dx_r}{dt} = \frac{d}{dt}\left(-\frac{1}{2}t^2 + 20t\right) = -t + 20 = 0 \longrightarrow t = 20\ s$$

$$* : t = 20\ s \Longrightarrow x_r(t=20) = -\frac{1}{2} \times 20^2 + 20 \times 20 = \frac{1}{2} \times 20^2 = 200\ m$$

$$v_g = \frac{dx_g}{dt} = \frac{d}{dt}\left(\frac{1}{2}t^2 - 40t + 950\right) = t - 40 = 0 \longrightarrow t = 40\ s$$

$** : t = 40\ s \Rightarrow x_g(t=40) = \frac{1}{2} \times 40^2 - 40 \times 40 + 950 = -\frac{1}{2} \times 40^2 \times 950 = -800 + 950 = 150\ m$

$x=150,\ v=0$ $\quad x=200$ $\quad x=0$ $\quad v=20\ km/h$ $\quad v=0$ $\quad x_0=950,\ v_0=-40\ km/h$

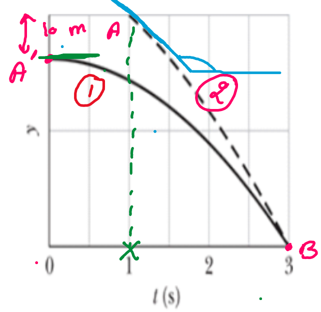
green train: $v^2 - v_0^2 = 2a(x - x_0)$ $\qquad v = ?\ ,\ x = 200\ m$

$v^2 - (-40)^2 = 2 \times 1 \times (\underbrace{200 - 950}_{-750}) \Rightarrow v^2 = 1600 - 1500 = 100 \Rightarrow v = \mp\sqrt{100} = -10\ m/s$

79 GO At time $t = 0$, a rock climber accidentally allows a piton to fall freely from a high point on the rock wall to the valley below him. Then, after a short delay, his climbing partner, who is 10 m higher on the wall, throws a piton downward. The positions y of the pitons versus t during the falling are given in Fig. 2.28. With what speed is the second piton thrown?

$y_A = 10 + y_A'$

$y_B = \frac{1}{2} a (t_B - t_A)^2 + v_A (t_B - t_A) + y_A$ $\quad *$



$A \rightarrow$ origin ; index: 0 ; $t_0 = 0$,

$B \rightarrow$ any point , without index

$y = \frac{1}{2} a t^2 + v_0 t + y_0$.

climber 2: $y_B = 0$, $t_B = 3\ s$, $t_A = 1\ s$, $a = -g$

$* : 0 = \frac{1}{\cancel{2}} \times (-g)(\underbrace{3-1}_{\cancel{4}\ 2})^2 + v_A (\underbrace{3-1}_{2}) + y_A \Rightarrow 2v_A + y_A - 2g = 0$ $\quad *$

climber 1 : $y_B = 0$, $t_B = 3S$, $t_A' = 0$, $v_A' = 0$, $a = -g$

$* : 0 = \frac{1}{2} \times (-g)(3-0)^2 + 0 + y_A' \rightarrow y_A' - \frac{9}{2} g = 0 \rightarrow y_A' = \frac{9}{2} g$

$\rightarrow y_A = 10 + \frac{9}{2} g \overset{*}{\Rightarrow} 2v_A + 10 + \frac{9}{2} g - 2g = 0 \rightarrow 2v_A + 10 + \frac{5}{2} g = 0$

$\rightarrow v_A = -5 - \frac{5}{4} \times 9.8 = -17.25 \rightarrow v_A \simeq -17\ m/s$