71 CALC In an arcade video game, a spot is programmed to move across the screen according to $x = 9.00t - 0.750t^3$, where x is distance in centimeters measured from the left edge of the screen and t is time in seconds. When the spot reaches a screen edge, at either $x = 0$ or $x = 15.0$ cm, t is reset to 0 and the spot starts moving again according to $x(t)$. (a) At what time after starting is the spot instantaneously at rest? (b) At what value of x does this occur? (c) What is the spot's acceleration (including sign) when this occurs? (d) Is it moving right or left just prior to coming to rest? (e) Just after? (f) At what time $t > 0$ does it first reach an edge of the screen?

$x = 9t - 0.75t^3$ * ; x: cm , t: s

a) $v = \frac{dx}{dt} = 9 - \underbrace{0.75}_{3/4} \times 3t^2 = 9 - \frac{9}{4}t^2$ *'

$v = 0 \Rightarrow 9 - \frac{9}{4}t^2 = 0 \Rightarrow \cancel{9} = \cancel{9}/4\, t^2 \Rightarrow t^2 = 4 \Rightarrow t = \mp 2 \longrightarrow$ $t = 2.00$ s

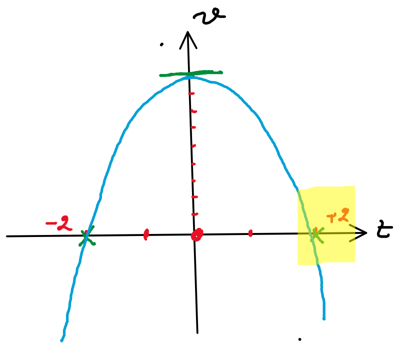
b) x : $x(t=2) = 9 \times 2 - \frac{3}{\cancel{4}} \times \cancel{2^3}^{2} = 18 - 6 =$ 12.0 cm

c) $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} \overset{*'}{=} \frac{d}{dt}\left(9 - \frac{9}{4}t^2\right) = -\frac{9}{4} \times 2t \Rightarrow a(t) = -\frac{9}{2}t$

$a(t=2) = -\frac{9}{2} \times 2 \longrightarrow a(t=2) =$ -9.00 cm/s^2

d,e) $v = 9 - \frac{9}{4}t^2$

$\frac{dv}{dt} = -\frac{9}{2}t = 0 \longrightarrow t = 0 \longrightarrow v(t=0) = 9 \leftarrow v_{max}$



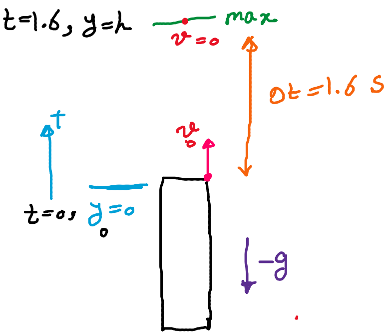
d) $t < 2$: $v > 0 \longrightarrow$ rightward

e) $t > 2$: $v < 0 \longrightarrow$ leftward

f) $x = 9t - \frac{3}{4}t^3 = 0 \longrightarrow 3t\left(3 - \frac{1}{4}t^2\right) = 0 \longrightarrow \begin{cases} t = 0 \\ 3 - \frac{t^2}{4} = 0 \longrightarrow t^2 = 12 \longrightarrow t = \mp 2\sqrt{3} \end{cases}$

$\longrightarrow t = 2\sqrt{3} \simeq$ 3.46 s

72 A rock is shot vertically upward from the edge of the top of a tall building. The rock reaches its maximum height above the top of the building 1.60 s after being shot. Then, after barely missing the edge of the building as it falls downward, the rock strikes the ground 6.00 s after it is launched. In SI units: (a) with what upward velocity is the rock shot, (b) what maximum height above the top of the building is reached by the rock, and (c) how tall is the building?



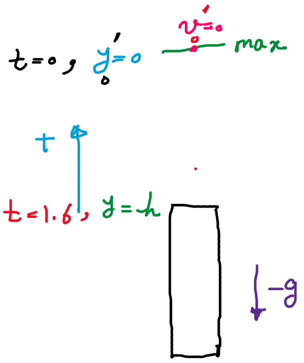
a) $v = -gt + v_0 \rightarrow 0 = -9.8 \times 1.6 + v_0 \rightarrow \boxed{v_0 = 15.7 \text{ m/s}}$

b) Solution 1 : $\cancel{v^2} - v_0^2 = -2g(y - \cancel{y_0})$

$-(15.7)^2 = -2 \times 9.8 \times (h - 0)$

$\Downarrow$

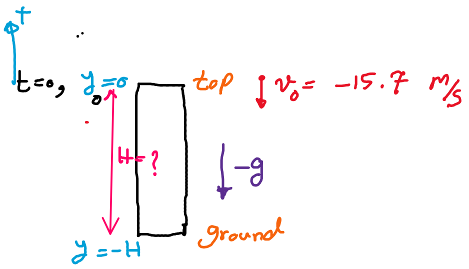
$h = \dfrac{(15.7)^2}{2 \times 9.8} \simeq \boxed{12.5 \text{ m}}$



b) Solution 2

$y = -\frac{1}{2}gt^2 + \cancel{v_0' t} + \cancel{y_0'}$

$-h = -\frac{1}{2} \times 9.8 \times (1.6)^2 \rightarrow \boxed{h \simeq 12.5 \text{ m}}$



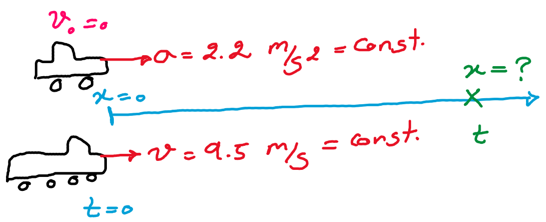
$t = 6 - 2 \times 1.6 = 2.8$ s

c) $y - \cancel{y_0} = -\frac{1}{2}gt^2 + v_0 t$

$-H = -\frac{1}{2} \times 9.8 \times (2.8)^2 - 15.7 \times 2.8$

$\rightarrow \boxed{H = 82.3 \text{ m}}$

73 GO At the instant the traffic light turns green, an automobile starts with a constant acceleration a of 2.2 m/s^2. At the same instant a truck, traveling with a constant speed of 9.5 m/s, overtakes and passes the automobile. (a) How far beyond the traffic signal will the automobile overtake the truck? (b) How fast will the automobile be traveling at that instant?



a) car : $x = \frac{1}{2}at^2 + \cancel{v_0 t} + \cancel{x_0}$

truck : $x = vt + \cancel{x_0}$ *

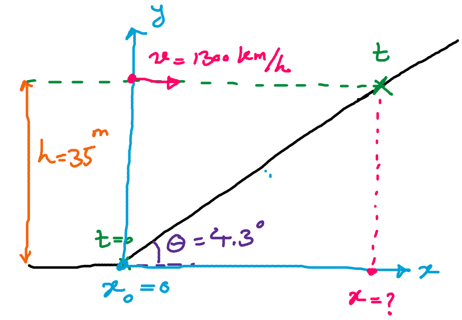
$x_{car} = x_{truck} \rightarrow \frac{1}{2}at^2 = vt \Rightarrow \frac{1}{2} \times 2.2\, t^2 = 9.5t \rightarrow 1.1t^2 - 9.5t = 0$

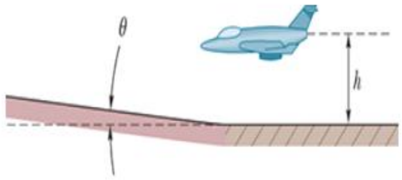
$\rightarrow t(1.1t - 9.5) = 0 \rightarrow \begin{cases} t = 0 \\ 1.1t - 9.5 = 0 \rightarrow t = 8.6 \text{ s} \end{cases}$

* : $t = 8.6 \rightarrow x = 9.5 \times 8.6 \simeq 82.04 \simeq \boxed{82 \text{ m}}$

b) $v = at + \cancel{v_0} \rightarrow v = 2.2 \times 8.6 = \boxed{19 \text{ m/s}}$

74 A pilot flies horizontally at 1300 km/h, at height $h = 35$ m above initially level ground. However, at time $t = 0$, the pilot begins to fly over ground sloping upward at angle $\theta = 4.3°$ (Fig. 2.26). If the pilot does not change the airplane's heading, at what time t does the plane strike the ground?



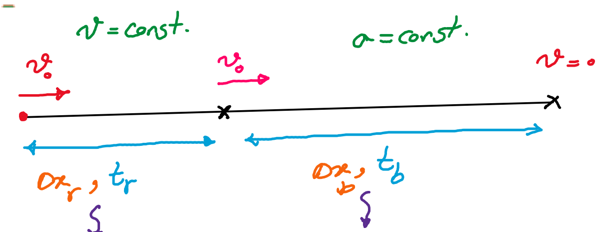


$y = \overset{\text{slope}}{\alpha} x \;;\; \alpha = \tan\theta \longrightarrow \overset{=35}{y} = \tan(4.3°)\, x \longrightarrow x = \dfrac{35}{\tan(4.3°)} = \underline{465.48 \text{ m}}$

airplane : $v = 1300 \text{ km/h} - \text{const.} \longrightarrow x = vt + \cancel{x_0} \longrightarrow t = \dfrac{465.48 \text{ m}}{1300 \times \dfrac{10^3 \text{ m}}{3600 \text{ s}}} \simeq 1.289$

$\Longrightarrow \boxed{t \simeq 1.3 \text{ s}}$

75 To stop a car, first you require a certain reaction time to begin braking; then the car slows at a constant rate. Suppose that the total distance moved by your car during these two phases is 56.7 m when its initial speed is 80.5 km/h, and 24.4 m when its initial speed is 48.3 km/h. What are (a) your reaction time and (b) the magnitude of the acceleration?



① : $\Delta x_r + \Delta x_b = 56.7^{\text{m}}$, $v_0 = 80.5 \text{ km/h} \times \frac{10}{36} = 22.4 \text{ m/s}$

② : $\Delta x_r + \Delta x_b = 24.4$, $v_0 = 24.4 \text{ km/h} \times \frac{10}{36} = 13.4 \text{ m/s}$

$\Delta x_r = v_0 t_r$

$\Delta x_b = \frac{1}{2} a t_b^2 + v_0 t_b \longrightarrow \Delta x_r + \Delta x_b = \frac{1}{2} a t_b^2 + v_0 t_b + v_0 t_r$

$\underset{0}{v} = a t_b + v_0 \Rightarrow t_b = \dfrac{-v_0}{a}$

$\Longrightarrow \Delta x_r + \Delta x_b = \underbrace{\frac{1}{2}\cancel{a} \times \frac{v_0^2}{a^{\cancel{2}}} + v_0 \times \frac{-v_0}{a}}_{-\frac{1}{2} v_0^2/a} + v_0 t_r \longrightarrow \Delta x_r + \Delta x_b = \dfrac{-v_0^2}{2a} + v_0 t_r$

① : $56.7 = \dfrac{-\overbrace{(22.4)^2}^{501.76}}{2a} + 22.4\, t_r$

② : $24.4 = \dfrac{-\overbrace{(13.4)^2}^{179.56}}{2a} + 13.4\, t_r$

$\longrightarrow \begin{cases} 22.4\, t_r - \dfrac{501.76}{2a} = 56.7 \;^{*} \xrightarrow{\times 13.4} \\ 13.4\, t_r - \dfrac{179.56}{2a} = 24.4 \xrightarrow{\times 22.4} \end{cases}$

$\longrightarrow \begin{cases} 22.4 \times 13.4\, t_r - \dfrac{501.76}{2a} \times 13.4 = 56.7 \times 13.4 \\ 13.4 \times 22.4\, t_r - \dfrac{179.56}{2a} \times 22.4 = 24.4 \times 22.4 \end{cases}$

⊖ : $0 + \frac{1}{a} \underbrace{\left\{ \frac{-501.76}{2} \times 13.4 + \frac{179.56}{2} \times 22.4 \right\}}_{-1350.72} = \underbrace{56.7 \times 13.4 - 24.4 \times 22.4}_{213.22}$

$\Longrightarrow \dfrac{-1350.72}{a} = 213.22 \Longrightarrow \boxed{a \simeq -6.3 \text{ m/s}^2} \xrightarrow{*} 22.4\, t_r - \dfrac{501.76}{2 \times (-6.3)} = 56.7 \longrightarrow \boxed{t_r \simeq 0.75 \text{ s}}$