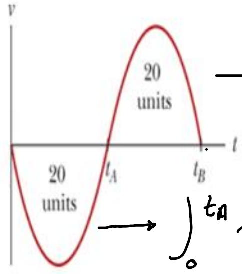
87 CALC *Velocity versus time.* Figure 2.32 gives the velocity v (m/s) versus time t (s) for a particle moving along an x axis. The area between the time axis and the plotted curve is given for the two portions of the graph. At $t = t_A$ (at one of the crossing points in the plotted figure), the particle's position is $x = 14$ m. What is its position at (a) $t = 0$ and (b) $t = t_B$?

$x(t_A) = 14\ m$

$x(t=0) = ?$ $\quad x(t=t_B) = ?$



$\longrightarrow \int_{t_A}^{t_B} v\,dt = +20$ (1)

$\longrightarrow \int_0^{t_A} v\,dt = -20$ (2)

$$v = \frac{dx}{dt} \longrightarrow \underbrace{x = \int v\,dt}$$

$$x(t_2) - x(t_1) = \int_{t_1}^{t_2} v\,dt$$

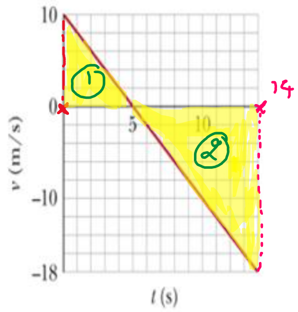
$\xrightarrow{(1)}$ $x(t_B) - \underbrace{x(t_A)}_{14} = \int_{t_A}^{t_B} v\,dt = 20 \longrightarrow \boxed{x(t_B) = 34}\ m$

$\xrightarrow{(2)}$ $\underbrace{x(t_A)}_{14} - x(0) = \int_0^{t_A} v\,dt = -20 \longrightarrow \boxed{x(t=0) = 34}\ m$

88 CALC *Hockey puck on frozen lake.* At time $t = 0$, a hockey puck is sent sliding over a frozen lake, directly into a strong wind. Figure 2.33 gives the velocity v of the puck versus time, as the puck moves along an x axis, starting at $x_0 = 0$. At $t = 14$ s, what is its coordinate?

$$v = \frac{dx}{dt} \longrightarrow \underbrace{x = \int v\,dt}$$

$$x(t_2) - x(t_1) = \int_{t_1}^{t_2} v\,dt$$



$x(t=14) = ?$

$x(t=0) = 0$

Solution 1

$$x(t=14) - x(t=0) = \underbrace{\int_{t=0}^{t=14} v\,dt} \quad *$$

$S_1 = \frac{1}{2} \times 5 \times 10 = 25$

$S_2 = \frac{1}{2} \times 9 \times (-18) = -81$

$\Big\} \longrightarrow S_1 + S_2 = -56 \longrightarrow \int_0^{14} v\,dt = -56$

$* \longrightarrow x(t=14) - 0 = -56 \longrightarrow \boxed{x(t=14) = -56\ m}$

Solution 2

$v = \alpha t + \beta \Longrightarrow \boxed{v = -2t + 10}$

$v(t=5) = 0 \Longrightarrow 0 = \alpha \times 5 + \beta \Longrightarrow \alpha = \frac{-\beta}{5} = \frac{-10}{5} \Longrightarrow \alpha = -2$

$v(t=0) = 10 \Longrightarrow 10 = \alpha \times 0 + \beta \longrightarrow \beta = 10$

$x(t_2) - x(t_1) = \underbrace{\int_{t_1}^{t_2} v\,dt}$; $\quad t_1 = 0$, $t_2 = 14$

$$\int_0^{14} (-2t+10)\,dt = \int_0^{14} (-2)t\,dt + \int_0^{14} 10\,dt = -2\int_0^{14} \underbrace{t\,dt}_{\frac{1}{2}t^2} + 10\int_0^{14} \underbrace{dt}_{t}$$

$$= -\cancel{2} \times \frac{1}{\cancel{2}} t^2 \Big|_0^{14} + 10t \Big|_0^{14} = -t^2 \Big|_0^{14} + 10t \Big|_0^{14} = \underbrace{-(14)^2}_{-196} - 0 + \underbrace{10 \times 14}_{140} - \underbrace{10 \times 0}_{0} = -56$$

$x(t=14) - \cancel{x(t=0)}^{0} = -56 \longrightarrow \boxed{x(t=14) = -56\ m}$

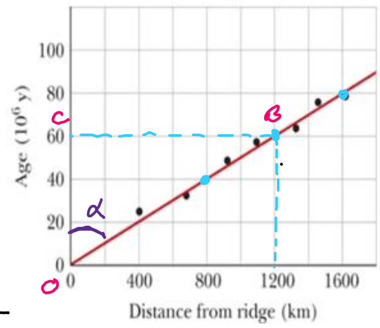
89 *Seafloor spread.* Figure 2.34 is a plot of the age of ancient seafloor material, in millions of years, against the distance from a certain ocean ridge. Seafloor material extruded from that ridge moves away from it at approximately uniform speed. What is that speed in centimeters per year?

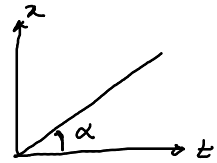
$$speed = \frac{distance}{time}$$

$$\overset{\triangle}{OBC} : \tan\alpha = \frac{\overline{BC}}{\overline{OC}} = \frac{\overset{20}{\cancel{1200}}\ km}{\cancel{60} \times 10^6\ year}$$

$$\rightarrow speed = 20 \times 10^{-6}\ \frac{km}{year} \rightarrow ?\ \frac{cm}{year}$$

$$speed = 20 \times 10^{-6} \times \frac{10^3 \times 10^2\ cm}{year} = \boxed{2\ cm/year}$$





$$\tan\alpha = \frac{x}{t}$$

90 *Braking, no reaction time.* Modern cars with a computer system using radar can eliminate the normal reaction time for a driver to recognize an upcoming danger and apply the brakes. For example, the system can detect the sudden stopping of a car in front of a driver by using radar signals that travel at the speed of light. Rapid processing then can almost immediately activate the braking. For a car traveling at $v = 31.3$ m/s (70 mi/h) and assuming a normal reaction time of 0.750 s, find the reduction in a car's stopping distance with such a computer system.

v = const.

see — brake — stop

reaction time = 0.750 s

$v = 31.3\ m/s$

$$distance = v.t = 31.3 \times 0.750 = 23.475 \simeq \boxed{23.5\ m}$$

91 *100 m dash.* The running event known as the 100 m dash consists of three stages. In the first, the runner accelerates to the maximum speed, which usually occurs at the 50 m to 70 m mark. That speed is then maintained until the last 10 m, when the runner slows. Consider three parts of the record-setting run by Usain Bolt in the 2008 Olympics: (a) from 10 m to 20 m, elapsed time of 1.02 s, (b) from 50 m to 60 m, elapsed time of 0.82 s, and (c) from 90 m to 100 m, elapsed time of 0.90 s. What was the average velocity for each part?

$x = 0$ 10 20 50 60 90 100

$\Delta t_1 = 1.02 s$ $\Delta t_2 = 0.82 s$ $\Delta t_3 = 0.90 s$

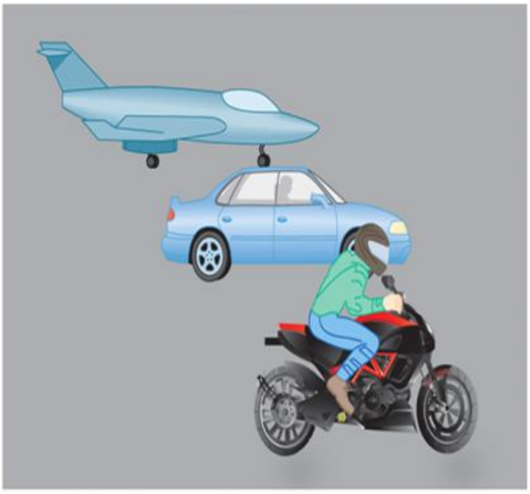
$$\bar{v} = \frac{\Delta x}{\Delta t}$$

a) $\bar{v}_1 = \frac{\Delta x_1}{\Delta t_1} = \frac{10}{1.02} = \boxed{9.80\ m/s}$

b) $\bar{v}_2 = \frac{\Delta x_2}{\Delta t_2} = \frac{10}{0.82} = \boxed{12.2\ m/s}$

c) $\bar{v}_3 = \frac{\Delta x_3}{\Delta t_3} = \frac{10}{0.90} = \boxed{11.1\ m/s}$

92 *Drag race of car and motorcycle.* A popular web video shows a jet airplane, a car, and a motorcycle racing from rest along a runway (Fig. 2.35). Initially the motorcycle takes the lead, but then the jet takes the lead, and finally the car blows past the motorcycle. Consider the motorcycle–car race. The motorcycle's constant acceleration $a_m = 8.40\ \text{m/s}^2$ is greater than the car's constant acceleration $a_c = 5.60\ \text{m/s}^2$, but the motorcycle has an upper limit of $v_m = 58.8$ m/s to its speed while the car has an upper limit of $v_c = 106$ m/s. Let the car and motorcycle race in the positive direction of an x axis, starting with their midpoints at $x = 0$ at $t = 0$. At what (a) time and (b) position are their midpoints again aligned?



$v_{oc} = 0$ $a_c = 5.60\ m/s^2$

$t = 7\ s$ $t = ?$

$x = 0, t = 0$ $x = 205.8$ $x = ?$

$a_m = 8.40\ m/s^2$ $v = v_m = 58.8\ m/s = \text{Const.}$

$v_{om} = 0$

Case 1 : both car and motorcycle are in the acceleration phase : X

$x = \frac{1}{2}at^2 + v_0 t + x_0 \longrightarrow$ Car: $x_c = \frac{1}{2}a_c t^2 + \cancel{v_{oc}} + \cancel{x_{oc}}$ (0, 0)

$t \le 7 \rightarrow$ motorcycle: $\overset{*}{x_m} = \frac{1}{2}a_m t^2 + \cancel{v_{om}} + \cancel{x_{om}}$ (0, 0)

$\Rightarrow x_c = x_m \Rightarrow \frac{1}{2}a_c t^2 = \frac{1}{2}a_m t^2$

Case 2 : motorcycle : $v = v_m = \text{Const.}$, Car: $a = \text{Const.}$

$v = at + v_0 \longrightarrow$ Car: $v_c = a_c t_c + \cancel{v_{oc}} \Rightarrow t_c = \frac{106}{5.60} \simeq 18.93\ s$

motorcycle: $v_m = a_m t_m + \cancel{v_{om}} \rightarrow t_m = \frac{58.8}{8.40} = 7\ s$

$\downarrow *$

$x_m = \frac{1}{2} \times 8.40 \times 7^2 = 205.8$

motorcycle : $\overset{*'}{x_m} = 205.8 + 58.8\,(t-7)$; $\underline{t \geq 7}$

car : $x_c = \frac{1}{2}a_c t^2 + \cancel{v_{oc}}t + \cancel{x_{oc}} = \frac{1}{2} \times 5.60\, t^2 = 2.8\, t^2$

$ax^2 + 2bx + c = 0$

$x = \frac{-b \mp \sqrt{b^2 - ac}}{a}$

$x_m = x_c \rightarrow 205.8 + 58.8\,(t-7) = 2.8t^2 \Rightarrow 2.8t^2 - 58.8t + 205.8 = 0$

$a = 2.8$, $b = -29.4$, $c = 205.8 \Rightarrow t = \frac{29.4 \mp \sqrt{(29.4)^2 - 205.8 \times 2.8}}{2.8} = \frac{29.4 \mp 16.97}{2.8}$

$\ominus$: $t = 4.44\ s \rightarrow$ unacceptable $(t < 7)$

$\oplus$: $t = 16.56\ s$ $\xrightarrow{*'}$ $x_m(t = 16.56) = 205.8 + 58.8\,(16.56 - 7) = 767.928 \simeq 768\ m = x_c(t = 16.56)$