14 E GO CALC An electron moving along the x axis has a position given by $x = 16te^{-t}$ m, where t is in seconds. How far is the electron from the origin when it momentarily stops?

$x = \underbrace{16t}_{A}\,\underbrace{\overset{a=-1}{e^{-t}}}_{B}$; x: m, t: s $\qquad v = 0 \rightarrow x - x_0 = ?$

$x_0 = x(t=0) = 0$

$\frac{d}{dt}(AB) = \frac{dA}{dt}B + A\frac{dB}{dt}$

$\frac{d}{dt}(e^{at}) = ae^{at}$

$v = \frac{dx}{dt} = 16e^{-t} + 16t\times(-1)\times e^{-t} = 16e^{-t} - 16te^{-t}$

$\rightarrow v = 16e^{-t}(1-t)$

$v = 0 \rightarrow e^{-t} = 0$ or $1 - t = 0$

unacceptable: $t \rightarrow \infty$ $\qquad t = 1 \rightarrow x(t=1) = 16 \times \underbrace{e^{-1}}_{\simeq 0.368} \Rightarrow \boxed{x \simeq 5.9 \text{ m}}$

$x = \infty \times e^{-\infty} = \infty \times 0$

15 E GO CALC (a) If a particle's position is given by $x = 4 - 12t + 3t^2$ (where t is in seconds and x is in meters), what is its velocity at $t = 1$ s? (b) Is it moving in the positive or negative direction of x just then? (c) What is its speed just then? (d) Is the speed increasing or decreasing just then? (Try answering the next two questions without further calculation.) (e) Is there ever an instant when the velocity is zero? If so, give the time t; if not, answer no. (f) Is there a time after $t = 3$ s when the particle is moving in the negative direction of x? If so, give the time t; if not, answer no.

$x = 4 - 12t + 3t^2$; x: m, t: s

a) $v = \frac{dx}{dt} = -12 + 3\times 2t \rightarrow v = -12 + 6t$

$v(t=1) = -12 + 6\times 1 = \boxed{-6 \text{ m/s}}$

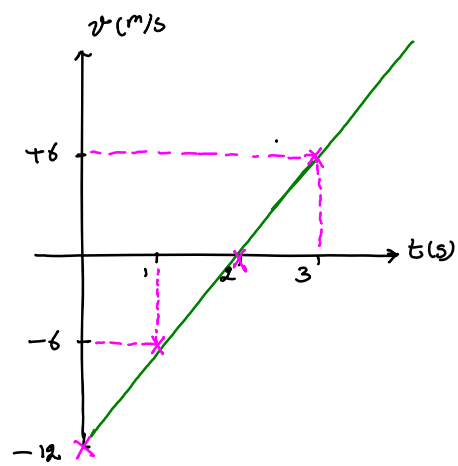
b) **negative direction**

c) $S = |v| = \boxed{6 \text{ m/s}}$

d) **Speed is decreasing**

e) $\boxed{t = 2} \rightarrow v = 0$

f) **No**



16 E CALC The position function $x(t)$ of a particle moving along an x axis is $x = 4.0 - 6.0t^2$, with x in meters and t in seconds. (a) At what time and (b) where does the particle (momentarily) stop? At what (c) negative time and (d) positive time does the particle pass through the origin? (e) Graph x versus t for the range -5 s to $+5$ s. (f) To shift the curve rightward on the graph, should we include the term $+20t$ or the term $-20t$ in $x(t)$? (g) Does that inclusion increase or decrease the value of x at which the particle momentarily stops?

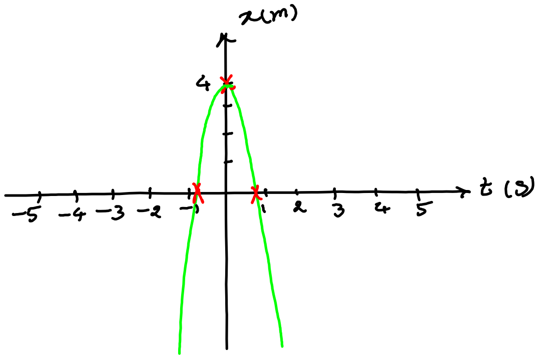
$x = 4.0 - 6.0t^2$ * ; x: m , t: s

a) $v = 0 \rightarrow v = \frac{dx}{dt} = -6\times 2t = -12t = 0 \rightarrow \boxed{t = 0}$

b) $x(t=0) = 4 - 0 = \boxed{4.0 \text{ m}}$

c, d) $x = 0 \xrightarrow{*} 4 - 6t^2 = 0 \rightarrow t^2 = \frac{4}{6} = \frac{2}{3} \Rightarrow t = \mp\sqrt{\frac{2}{3}}\text{ s} = \boxed{\mp 0.82 \text{ s}}$

e)



$v = -12t \longrightarrow a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2} = -12 < 0$

$x = 4 - 6t^2$

$x(t = \mp 1) = 4 - 6 = -2\ m$

$x(t = \mp 2) = 4 - 6 \times 4 = -20\ m$

$x(t = \mp 3) = 4 - 6 \times 9 = -50\ m$

$x(t = \mp 4) = 4 - 6 \times 16 = -92\ m$

$x(t = \mp 5) = 4 - 6 \times 25 = -146\ m$

f) $+20t$

g) $x = 4 - 6t^2 + 20t$

$v = 0 \longrightarrow v = \dfrac{dx}{dt} = -12t + 20 = 0 \longrightarrow t = \dfrac{20}{12} = \dfrac{5}{3} \simeq 1.67\ s$

$x(t = 1.67) = 4 - 6 \times (1.67)^2 + 20 \times 1.67 \simeq 26\ m$

17 M CALC The position of a particle moving along the x axis is given in centimeters by $x = 9.75 + 1.50t^3$, where t is in seconds. Calculate (a) the average velocity during the time interval $t = 2.00$ s to $t = 3.00$ s; (b) the instantaneous velocity at $t = 2.00$ s; (c) the instantaneous velocity at $t = 3.00$ s; (d) the instantaneous velocity at $t = 2.50$ s; and (e) the instantaneous velocity when the particle is midway between its positions at $t = 2.00$ s and $t = 3.00$ s. (f) Graph x versus t and indicate your answers graphically.

$x = 9.75 + 1.5t^3$ * ; x: cm , t: s

a) $\bar{v}_{t=2,3} = ?$ $\qquad \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{x(t=3) - x(t=2)}{3 - 2 = 1}$

* : $t = 3 \longrightarrow x(t=3) = 9.75 + 1.5 \times 3^3 \simeq 50.25$ cm $\longrightarrow \bar{v} = 50.25 - 21.75 \simeq 28.5$ cm/s

* : $t = 2 \longrightarrow x(t=2) = 9.75 + 1.5 \times 2^3 \simeq 21.75$ cm

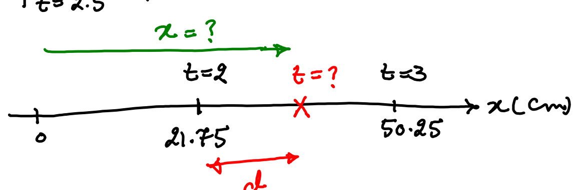
b) $v|_{t=2} = ?$ $\qquad v = \dfrac{dx}{dt} \overset{*}{=} 1.5 \times 3t^2 = 4.5t^2$ **

$v(t=2) = 4.5 \times 2^2 \simeq 18.0$ cm/s

c) $v|_{t=3} = ?$ $\qquad$ ** : $t = 3 \longrightarrow v(t=3) = 4.5 \times 3^2 \simeq 40.5$ cm/s

d) $v|_{t=2.5} = ?$ $\qquad$ ** : $t = 2.5 \longrightarrow v(t=2.5) = 4.5 \times (2.5)^2 \simeq 28.1$ cm/s

e)



$d = \dfrac{50.25 - 21.75}{2} = 14.25$ cm

$\longrightarrow x = 21.75 + 14.25 = 36$ cm

* : $\overset{36}{x} = 9.75 + 1.50t^3 \longrightarrow 1.5t^3 = 36 - 9.75 = 26.25 \longrightarrow t^3 = \dfrac{26.25}{1.5} = 17.5$

$\longrightarrow t = \sqrt[3]{17.5} \simeq 2.596\ s$

$$**: \ t = 2.596 \longrightarrow v(t=2.596) = 4.5 \times (2.596)^2 \simeq \boxed{30.3 \ \text{cm/s}}$$

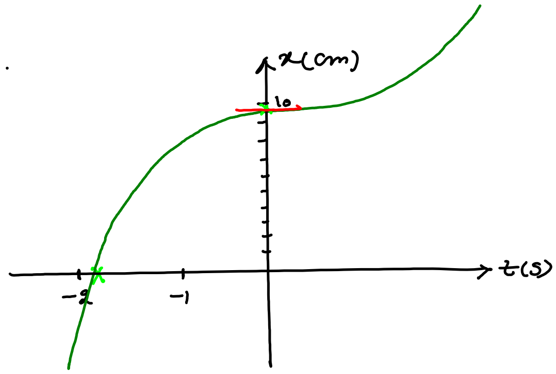
f) $x = 9.75 + 1.5\,t^3$

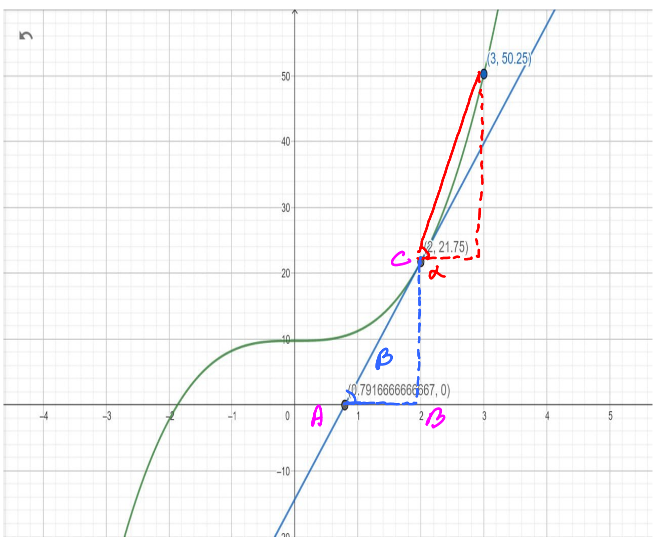
$t=0 \longrightarrow x = 9.75$ cm

$x=0 \longrightarrow 9.75 + 1.5\,t^3 = 0 \longrightarrow t^3 = \dfrac{-9.75}{1.5} \simeq -6.5 \longrightarrow t \simeq -1.87$

$\dfrac{dx}{dt} = 0 \longrightarrow 4.5\,t^2 = 0 \implies t = 0$

$\underbrace{\dfrac{d^2x}{dt^2}}_{=a} = 9t \Big|_{t=0} = 0 \longrightarrow \begin{cases} t<0 \ : \ a<0 \\ t>0 \ : \ a>0 \end{cases}$





$\bar{v} = \tan\alpha = \dfrac{50.25 - 21.75}{3-2} = 28.5$ cm/s

$\tan\beta = \dfrac{\overline{BC}}{\overline{AB}} = \dfrac{21.75}{2-0.792 = 1.208} \simeq 18$ cm/s

$\downarrow$

$v(t=2)$