1 E While driving a car at 90 km/h, how far do you move while your eyes shut for 0.50 s during a hard sneeze?

$v = 90\ km/h \rightarrow$ assumed constant

$t = 0.50\ m/s$

$$v = 90\ km/h = \cancel{90} \times \frac{10^3\ m}{\cancel{3600}\ s} = 25\ m/s$$

$$d = vt \implies d = 25 \times 0.50 = 12{,}5 \simeq \boxed{13\ m/s}$$

2 E Compute your average velocity in the following two cases: (a) You walk 73.2 m at a speed of 1.22 m/s and then run 73.2 m at a speed of 3.05 m/s along a straight track. (b) You walk for 1.00 min at a speed of 1.22 m/s and then run for 1.00 min at 3.05 m/s along a straight track. (c) Graph x versus t for both cases and indicate how the average velocity is found on the graph.

a) $d_1 = 73.2\ m$ $\quad d_2 = 73.2\ m$

$x_i = 0$ (origin) $\quad v_1 = 1.22\ m/s \quad v_2 = 3.05\ m/s \quad x_f = 73.2 + 73.2$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$ final, initial

$$v = d/t \rightarrow t_1 = \frac{d_1}{v_1} = \frac{73.2}{1.22} = 60\ s$$

$$t_2 = \frac{d_2}{v_2} = \frac{73.2}{3.05} = 24\ s$$

$$\implies \Delta t = t_1 + t_2 = 60 + 24 = 84\ s$$

$$\rightarrow \bar{v} = \frac{2 \times 73{,}2 - 0}{84} \rightarrow \boxed{\bar{v} \simeq 1{,}47\ m/s}$$

b) $t_1 = 1\ min = 60\ s \quad t_2 = 1\ min = 60\ s$

$x_i = 0 \quad v_1 = 1.22\ m/s \quad v_2 = 3.05\ m/s \quad x_f = ?$

$d_1 \quad d_2$

$\Delta t = t_1 + t_2 = 2 \times 60 = 120\ s$

$d_1 = v_1 t_1 = 1.22 \times 60 = 73.2\ m$

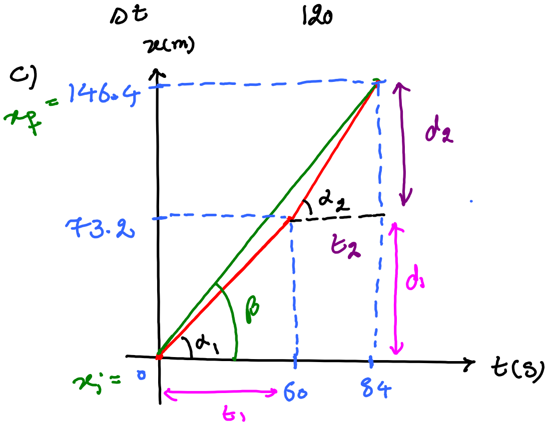
$d_2 = v_2 t_2 = 3.05 \times 60 = 183\ m$

$\rightarrow \Delta x = x_f - \cancel{x_i} = d_1 + d_2 = 60 \times (1.22 + 3.05)$

$\implies \Delta x = 60 \times 4.27 = 256.2\ m$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{256.2}{120} = 2.135 \simeq \boxed{2.14\ m/s}$$

c)

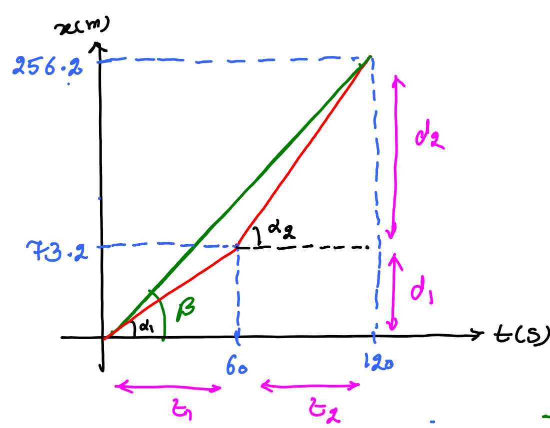


part a

$$\tan\alpha_1 = v_1 = \frac{d_1}{t_1}$$

$$\tan\alpha_2 = v_2 = \frac{d_2}{t_2}$$

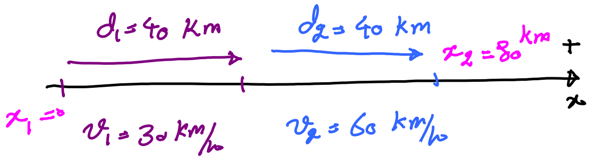
$$\tan\beta = \frac{d_1 + d_2}{t_1 + t_2} = \bar{v}$$



part b

$$\tan\beta = \frac{d_1 + d_2}{t_1 + t_2} = \bar{v}$$

3 E SSM An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during the full 80 km trip? (Assume that it moves in the positive x direction.) (b) What is the average speed? (c) Graph x versus t and indicate how the average velocity is found on the graph.



$$\Delta x = x_2 - x_1 = d_1 + d_2$$

$$\Delta t = t_1 + t_2$$

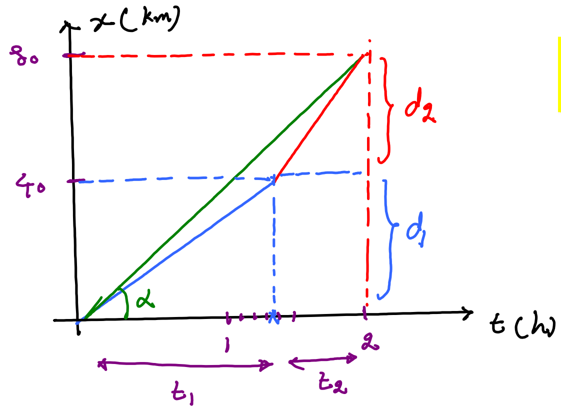
a) $\bar{v} = ?$ $\quad \bar{v} = \dfrac{\Delta x}{\Delta t}$ ⟶ displacement $\Rightarrow \boxed{\bar{v}} = \dfrac{d_1 + d_2}{t_1 + t_2} = \dfrac{40 + 40}{4/3 + 2/3} = \dfrac{80}{2} = \boxed{40 \text{ km/h}}$

$$v_1 = \frac{d_1}{t_1} \Rightarrow t_1 = \frac{d_1}{v_1} = \frac{40}{30} = \frac{4}{3} \text{ h}$$

$$t_2 = \frac{d_2}{v_2} = \frac{40}{60} = \frac{2}{3} \text{ h}$$

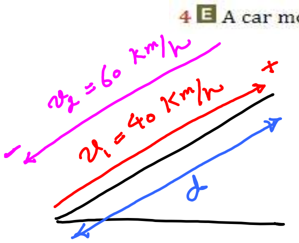
b) $\boxed{\text{average speed}}$ = average velocity = $\boxed{40 \text{ km/h}}$

c)



$$\boxed{\tan\alpha = \bar{v} = \frac{d_1 + d_2}{t_1 + t_2}}$$

4 E A car moves uphill at 40 km/h and then back downhill at 60 km/h. What is the average speed for the round trip?



$$S_{avg} = \frac{\text{total distance}}{\text{total time}} \quad \text{total time} = t_1 + t_2$$

total distance $= 2d$

$$v_1 = \frac{d_1}{t_1} \to t_1 = \frac{d}{40}$$

$$t_2 = \frac{d}{60}$$

$$\Rightarrow S_{avg} = \frac{2d}{d/40 + d/60} = \frac{2\cancel{d}}{\cancel{d}(1/40 + 1/60)} = \frac{3+2}{120} = \frac{5}{120} = \frac{1}{24}$$

$$\Rightarrow \boxed{S_{avg}} = \frac{2}{\frac{1}{24}} = \boxed{48 \text{ km/h}}$$

$\bar{v} = \dfrac{\Delta x}{\Delta t}$, Δx ⟶ displacement $= d + (-d) = 0$ $\Rightarrow \bar{v} = 0$

5 E CALC SSM The position of an object moving along an x axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds. Find the position of the object at the following values of t: (a) 1 s, (b) 2 s, (c) 3 s, and (d) 4 s. (e) What is the object's displacement between $t = 0$ and $t = 4$ s? (f) What is its average velocity for the time interval from $t = 2$ s to $t = 4$ s? (g) Graph x versus t for $0 \le t \le 4$ s and indicate how the answer for (f) can be found on the graph.

$x = 3t - 4t^2 + t^3$ *

a) $t = 1$ s ⟶ $\boxed{x(t=1)} = 3\times1 - 4\times1 + 1 = \boxed{0 \text{ m}}$

b) $t = 2$ s ⟶ $\boxed{x(t=2)} = 3\times2 - 4\times4 + 8 = \boxed{-2 \text{ m}}$

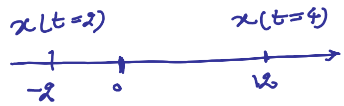
c) $t = 3$ s ⟶ $\boxed{x(t=3)} = 3\times3 - 4\times9 + 27 = \boxed{0 \text{ m}}$

d) $t = 4s \longrightarrow \boxed{x(t=4)} = 3\times 4 - 4\times 16 + 64 \boxed{= 12\ m}$

e) $\Delta x = x_f - x_i = \underbrace{x(t=4)}_{12} - \underbrace{x(t=0)}_{0} \longrightarrow \boxed{\Delta x = 12\ m}$

Final ↙ ↘ initial

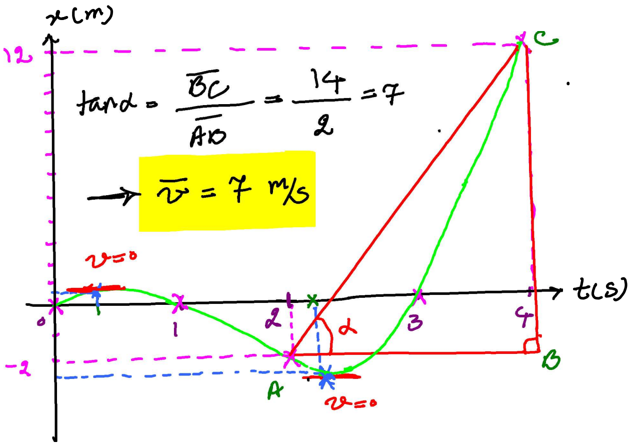
$* :\ t=0 \longrightarrow x(t=0) = 0$



f) $\boxed{\bar{v} =} \dfrac{\Delta x}{\Delta t} = \dfrac{\overbrace{x(t=4)}^{12} - \overbrace{x(t=2)}^{-2}}{4-2} = \boxed{7\ m/s}$

$x = 3t - 4t^2 + t^3$

g)



$\dfrac{dx}{dt} = 0 \longrightarrow \dfrac{dx}{dt} = 3 - 8t + 3t^2 = 0$

$3t^2 - 8t + 3 = 0 \longrightarrow a=3\ ;\ b=-4\ ,\ c=3$

$ax^2 + 2bx + c = 0 \longrightarrow x = \dfrac{-b \mp \sqrt{b^2 - ac}}{a}$

$\Rightarrow t = \dfrac{+4 \mp \sqrt{16-9}}{3} = \dfrac{4 \mp \sqrt{7}}{3}$ $\quad \sqrt{7} \approx 2{,}6$

$\Rightarrow \oplus :\ t = 2{,}22\ s$ (min) $\qquad \ominus :\ t = 0{,}45\ s$ (max)

$\dfrac{d^2x}{dt^2} = -8 + 6t = 0 \longrightarrow t = \dfrac{8}{6} = \dfrac{4}{3} \simeq 1{,}33\ s$

$t < 1{,}33 \longrightarrow$ example: $t=1 \longrightarrow \left.\dfrac{d^2x}{dt^2}\right|_{t=1} = -8 + 6\times 1 = -2 < 0 \longrightarrow$ concave down

$t > 1{,}33 \longrightarrow$ example : $t=2 \longrightarrow \left.\dfrac{d^2x}{dt^2}\right|_{t=2} = -8 \times 6\times 2 = +4 > 0 \longrightarrow$ concave up

$* \longrightarrow x(t=2{,}22) = 3\times 2{,}22 - 4\times (2{,}22)^2 + (2{,}22)^3 \simeq -2{,}11\ m$

$* \longrightarrow x(t=0{,}45) = 3\times 0{,}45 - 4\times (0{,}45)^2 + (0{,}45)^3 \simeq 0{,}63\ m$

6 E BIO The 1992 world speed record for a bicycle (human-powered vehicle) was set by Chris Huber. His time through the measured 200 m stretch was a sizzling 6.509 s, at which he commented, "Cogito ergo zoom!" (I think, therefore I go fast!). In 2001, Sam Whittingham beat Huber's record by 19.0 km/h. What was Whittingham's time through the 200 m?

$d = 200\ m$

Chris : ① $\qquad t_1 = 6{,}509\ s$

Sam : ② $\qquad t_2 = ? \qquad v_2 = v_1 + 19\ km/h$

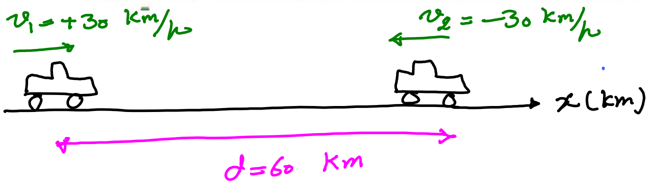
$19 \times \dfrac{10^3\ m}{3600\ s} \simeq 5{,}28\ m/s$

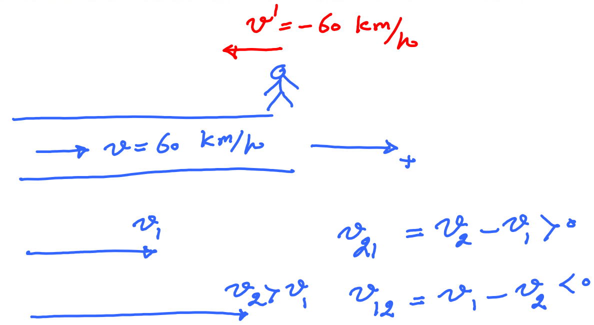
$v_1 = \dfrac{d}{t_1} \longrightarrow v_1 = \dfrac{200}{6{,}509} \simeq 30{,}73\ m/s$

$\longrightarrow v_2 = 30{,}73 + 5{,}28 \simeq 36\ m/s$

$v_2 = \dfrac{d}{t_2} \longrightarrow t_2 = \dfrac{200}{36} \simeq \boxed{5{,}56\ s}$

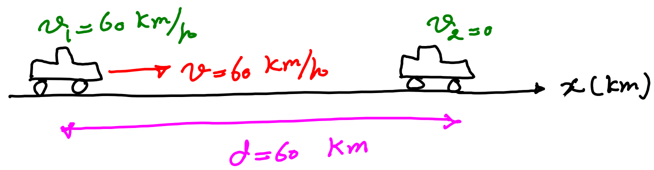
7 M Two trains, each having a speed of 30 km/h, are headed at each other on the same straight track. A bird that can fly 60 km/h flies off the front of one train when they are 60 km apart and heads directly for the other train. On reaching the other train, the (crazy) bird flies directly back to the first train, and so forth. What is the total distance the bird travels before the trains collide?





$v_{12} = v_1 - v_2 = +30 - (-30) = 60$ km/h

if $v_2 = v_1 \rightarrow v_{21} = 0$



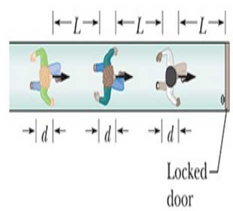
$v_1 = \frac{d}{t_1} \rightarrow t_1 = \frac{60}{60} = 1$ h $\rightarrow$ distance traveled by the bird $= 60 \times 1 = 60$ km

8 M FCP GO *Panic escape.* Figure 2.9 shows a general situation in which a stream of people attempt to escape through an exit door that turns out to be locked. The people move toward the door at speed $v_s = 3.50$ m/s, are each $d = 0.25$ m in depth, and are separated by $L = 1.75$ m. The arrangement in Fig. 2.9 occurs at time $t = 0$. (a) At what average rate does the layer of people at the door increase? (b) At what time does the layer's depth reach 5.0 m? (The answers reveal how quickly such a situation becomes dangerous.)

$v_s = 3{,}50$ m/s

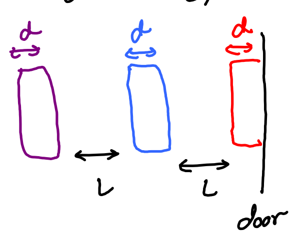
$d = 0{,}25$ m/s

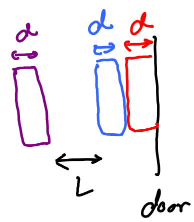
$L = 1{,}75$ m/s



a) $t = \frac{L}{v} = \frac{1{,}75}{3{,}50} = 0{,}50$ s

$t = 0{,}50 + 0{,}50 = 1{,}0$ s





Arrival rate of people = 2 People/s

Rate of depth increase $= 2 \times d = 0{,}50$ m/s

b) $D = 5{,}0$ m $\rightarrow$

s	m
1	0,50
x	5,0

$\rightarrow t = \frac{5{,}0}{0{,}50} = 10$ s

9 M BIO In 1 km races, runner 1 on track 1 (with time 2 min, 27.95 s) appears to be faster than runner 2 on track 2 (2 min, 28.15 s). However, length L_2 of track 2 might be slightly greater than length L_1 of track 1. How large can $L_2 - L_1$ be for us still to conclude that runner 1 is faster?

$t_1 = 2$ min, $27{,}95$ s $= 147{,}95$ s

$t_2 = 2$ min, $28{,}15$ s $= 148{,}15$ s

$v_1 = \frac{L_1}{147{,}95}$ $\qquad v_2 = \frac{L_2}{148{,}15}$

$v_1 > v_2 \rightarrow \frac{L_1}{147{,}95} > \frac{L_2}{148{,}15} \rightarrow L_2 < \underbrace{\frac{148{,}15}{147{,}95}}_{\simeq 1{,}00135} L_1$

if $L_1 = 1$ km $\Rightarrow L_2 < 1{,}00135$ km

$\rightarrow L_2 - L_1 < 1{,}00135 - 1 \simeq 0{,}00135$ km $\simeq 1{,}4$ m

$\rightarrow$ if $L_2 - L_1 < 1{,}4$ m or $L_2 < 1{,}4 + L_1 \Rightarrow$ runner 1 is faster

10 M FCP To set a speed record in a measured (straight-line) distance d, a race car must be driven first in one direction (in time t_1) and then in the opposite direction (in time t_2). (a) To eliminate the effects of the wind and obtain the car's speed v_c in a windless situation, should we find the average of d/t_1 and d/t_2 (method 1) or should we divide d by the average of t_1 and t_2? (b) What is the fractional difference in the two methods when a steady wind blows along the car's route and the ratio of the wind speed v_w to the car's speed v_c is 0.0240?

d $\quad \rightarrow v_w$

$v_c \rightarrow \; + \; : t_1$

$v_c \leftarrow \; - \; : t_2$

a) method 1 : $v_c = \dfrac{d/t_1 + d/t_2}{2}$ ✓

method 2 : $v_c' = \dfrac{d}{\frac{t_1 + t_2}{2}}$

$$v_+ = v_c + v_w \; ; \; t_1 = \frac{d}{v_+} \rightarrow t_1 = \frac{d}{v_c + v_w} \rightarrow v_c + v_w = \frac{d}{t_1}$$

$$v_- = v_c - v_w \; ; \; t_2 = \frac{d}{v_-} \rightarrow t_2 = \frac{d}{v_c - v_w} \; * \rightarrow v_c - v_w = \frac{d}{t_2}$$

$\oplus \Rightarrow$

$$2v_c = \frac{d}{t_1} + \frac{d}{t_2} \Rightarrow \boxed{v_c = \frac{d/t_1 + d/t_2}{2}}$$

b) $\dfrac{v_w}{v_c} = 0.0240$ $\qquad \left| \dfrac{v_c - v_c'}{v_c} \right| = ?$ $= A$ $\qquad A = \left| 1 - \dfrac{v_c'}{v_c} \right|$?

$$v_c' = \frac{d}{\frac{t_1 + t_2}{2}} = \frac{2d}{t_1 + t_2} \overset{*}{=} \frac{2d}{\frac{d}{v_c + v_w} + \frac{d}{v_c - v_w}} = \frac{\cancel{2d}}{\cancel{d}\left\{ \underbrace{\frac{1}{v_c + v_w} + \frac{1}{v_c - v_w}} \right\}}$$

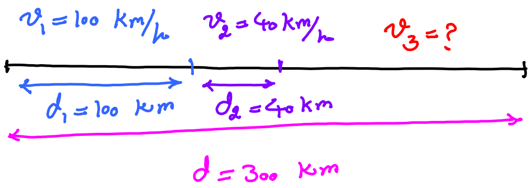
$$\frac{v_c - \cancel{v_w} + v_c + \cancel{v_w}}{(v_c + v_w)(v_c - v_w)} = \frac{\cancel{2}v_c}{v_c^2 - v_w^2}$$

$$\Rightarrow v_c' = \frac{1}{\frac{v_c}{v_c^2 - v_w^2}} = \frac{v_c^2 - v_w^2}{v_c} = v_c - \frac{v_w^2}{v_c} = v_c\left\{ 1 - \underline{\frac{v_w^2}{v_c^2}} \right\}$$

$$\left(\frac{v_w}{v_c}\right)^2 = (0.0240)^2$$

$$\Rightarrow \frac{v_c'}{v_c} = 1 - (0.0240)^2 \Rightarrow A = \left| 1 - \left(\frac{v_c'}{v_c}\right)^2 \right| = (0.0240)^2 \simeq \boxed{5{,}76 \times 10^{-4}}$$

11 M GO You are to drive 300 km to an interview. The interview is at 11:15 a.m. You plan to drive at 100 km/h, so you leave at 8:00 a.m. to allow some extra time. You drive at that speed for the first 100 km, but then construction work forces you to slow to 40 km/h for 40 km. What would be the least speed needed for the rest of the trip to arrive in time for the interview?



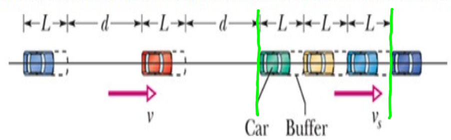
$v = \frac{d}{t} \rightarrow t_1 = \frac{d_1}{v_1} = \frac{100}{100} = 1$ h

$t_2 = \frac{d_2}{v_2} = \frac{40}{40} = 1$ h

8.00 AM leave $\longrightarrow$ 11:15 AM interview $\rightarrow$ $t = 3$h, $\underbrace{15 \text{ min}}_{1/4\text{ h} = 0{,}25\text{ h}} = 3{,}25$ h $\rightarrow t_3 = 3{,}25 - (t_1 + t_2) = 1{,}25$ h

$d_3 = 300 - (d_1 + d_2) = 300 - 140 = 160$ Km $\rightarrow v_3 = \frac{d_3}{t_3} = \frac{160}{1{,}25} = 128$ Km/h

12 H FCP *Traffic shock wave.* An abrupt slowdown in concentrated traffic can travel as a pulse, termed a *shock wave*, along the line of cars, either downstream (in the traffic direction) or upstream, or it can be stationary. Figure 2.10 shows a uniformly spaced line of cars moving at speed $v = 25.0$ m/s toward a uniformly spaced line of slow cars moving at speed $v_s = 5.00$ m/s. Assume that each faster car adds length $L = 12.0$ m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant. (a) For what separation distance d between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?

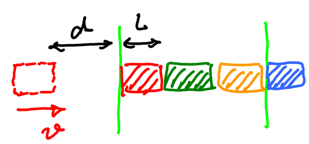


$t = 0$

$v_s = 5.00$ m/s

$v = 25.0$ m

$L = 12.0$ m



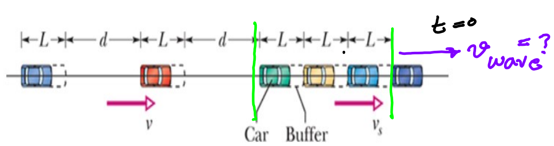
green : $t, L, v_s \rightarrow t = \frac{L}{v_s}$

red : $t, d+L, v \rightarrow t = \frac{d+L}{v}$

$\rightarrow \frac{L}{v_s} = \frac{d+L}{v} \rightarrow \frac{12.0}{5.\cancel{00}} = \frac{d+12.0}{\cancel{25.0}\ 5.}$

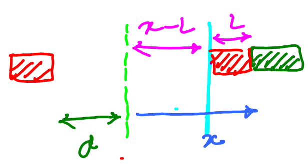
$\rightarrow \underbrace{5.0 \times 12.0}_{60.} = d + 12.0 \rightarrow d = 60.0 - 12.0 = 48.0$ m

b)



$d = 2 \times 48.0 = 96.0$ m

$t > 0$



green : $v_s, t, x \rightarrow t = \frac{x}{v_s}$ *

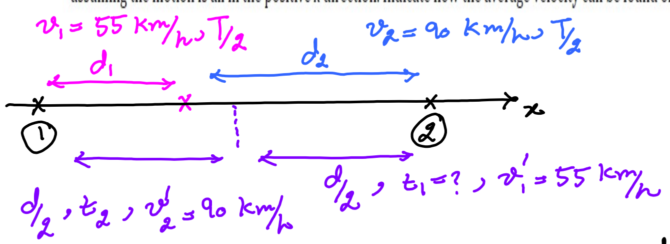
red : $v, t, d+x \rightarrow t = \frac{d+x}{v}$

$\rightarrow \frac{x}{v_s} = \frac{d+x}{v} \rightarrow \frac{x}{\cancel{5}} = \frac{96+x}{\cancel{25}\ 5}$

$\rightarrow 5x = 96 + x \rightarrow 4x = 96 \rightarrow x = 24.0$ m

* : $t = \frac{24.0}{5.00} = 4.80$ s $\rightarrow v_{wave} = \frac{x - L}{t} = \frac{24 - 12}{4.8} = 2.50$ m/s $\rightarrow$ downstream

13 H You drive on Interstate 10 from San Antonio to Houston, half the *time* at 55 km/h and the other half at 90 km/h. On the way back you travel half the *distance* at 55 km/h and the other half at 90 km/h. What is your average speed (a) from San Antonio to Houston, (b) from Houston back to San Antonio, and (c) for the entire trip? (d) What is your average velocity for the entire trip? (e) Sketch *x* versus *t* for (a), assuming the motion is all in the positive *x* direction. Indicate how the average velocity can be found on the sketch.



a) $S_{avg} = \dfrac{\text{distance}}{\text{time}} = \dfrac{d_1 + d_2}{2 \times T/2}$ $\qquad d_1 = v_1 \times T/2$, $\quad d_2 = v_2 \times T/2$

$\rightarrow S_{avg}^{(I)} = \dfrac{v_1 \times \cancel{T/2} + v_2 \times \cancel{T/2}}{\cancel{T}} = \frac{1}{2}(v_1 + v_2) = \frac{1}{2}(55 + 90) = 72.5 \simeq \boxed{73 \text{ km/h}}$

b) $S_{avg} = \dfrac{\text{distance}}{\text{time}} = \dfrac{2 \times d/2}{t_1 + t_2}$ $\qquad t_1 = \dfrac{d/2}{v_1'} = \dfrac{d}{2v_1'}$, $\quad t_2 = \dfrac{d/2}{v_2'} = \dfrac{d}{2v_2'}$

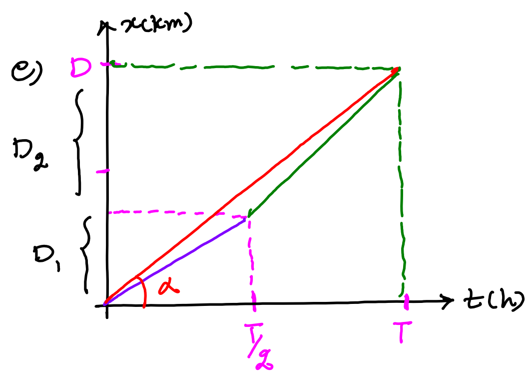
$\Rightarrow S_{avg}^{(II)} = \dfrac{\cancel{d}}{\dfrac{\cancel{d}}{2v_1'} + \dfrac{\cancel{d}}{2v_2'}} = \dfrac{2v_1' v_2'}{v_1' + v_2'} = \dfrac{2 \times 55 \times 90}{55 + 90} \simeq 68.3 \simeq \boxed{68 \text{ km/h}}$

c) $S_{avg} = \dfrac{\text{distance}}{\text{time}} = \dfrac{2d}{t_{1\to 2} + t_{2\to 1}}$ $\qquad t_{1\to 2} = \dfrac{d}{S_{avg}^{(I)}} = \dfrac{d}{72.5}$, $\quad t_{2\to 1} = \dfrac{d}{S_{avg}^{(II)}} = \dfrac{d}{68.3}$

$\rightarrow S_{avg} = \dfrac{2\cancel{d}}{\dfrac{\cancel{d}}{72.5} + \dfrac{\cancel{d}}{68.3}} = \dfrac{2 \times 72.5 \times 68.3}{68.3 + 72.5} = \dfrac{9903.5}{140.8} \simeq 70.3 \simeq \boxed{70 \text{ km/h}}$

d) $\Delta x = x_f - x_i \; ; \; x_f = x_i \rightarrow \Delta x = 0 \rightarrow \boxed{\bar{v} = 0}$

e)



$\tan\alpha = \dfrac{D}{T} = \frac{1}{2} \times 145 = 72.5 \simeq \boxed{73 \text{ km/h}}$

$\left.\begin{array}{l} D_1 = 55 \times T/2 \\ D_2 = 90 \times T/2 \end{array}\right\} \Rightarrow D = 145 \times T/2$