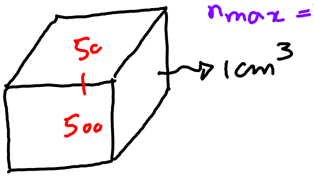
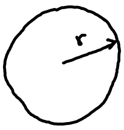
26 M One cubic centimeter of a typical cumulus cloud contains 50 to 500 water drops, which have a typical radius of 10 μm. For that range, give the lower value and the higher value, respectively, for the following. (a) How many cubic meters of water are in a cylindrical cumulus cloud of height 3.0 km and radius 1.0 km? (b) How many 1-liter pop bottles would that water fill? (c) Water has a density of 1000 kg/m^3. How much mass does the water in the cloud have?

$n_{min} = 50$

$n_{max} = 500$





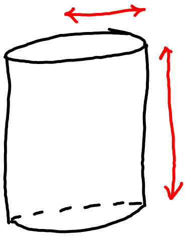
$r = 10\ \mu m = 10 \times 10^{-6}\ m = 10^{-5}\ m$

1 water drop : $v = \frac{4}{3}\pi r^3 = \frac{4}{3} \times 3{,}14 \times (10^{-5})^3$

$\simeq 4{,}2 \times 10^{-15}\ m$

$R = 1{,}0\ km = 1{,}0 \times 10^3\ m$

a)



$H = 3{,}0\ km = 3{,}0 \times 10^3\ m$

$V_{cloud} = \pi R^2 H$

$= 3{,}14 \times (1{,}0 \times 10^3)^2 \times 3{,}0 \times 10^3$

$= 9{,}42 \times 10^9\ \underbrace{m^3}_{(10^2\ cm)^3} = 9{,}4 \times 10^{15}\ cm^3$

$N_{min} = V \cdot n_{min} = 9{,}4 \times 10^{15} \times 50 = \underbrace{470}_{4{,}7 \times 10^2} \times 10^{15} \Rightarrow N_{min} = 4{,}7 \times 10^{17}$

$N_{max} = V \cdot n_{max}$ $\qquad N_{max} = 4{,}7 \times 10^{18}$

$\rightarrow V'_{min} = N_{min} \cdot v = 4{,}7 \times 10^{17} \times 4{,}2 \times 10^{-15} = \underbrace{19{,}47}_{20} \times 10^2 = 2 \times 10^3\ m^3$

$V'_{max} = N_{max} \cdot v = \dots$ $\qquad V'_{max} = 2 \times 10^4\ m^3$

b) $1\ liter = 10^3\ cc = 10^3\ \underbrace{cm^3}_{(10^{-2}\ m)^3} = 10^3 \times 10^{-6} = 10^{-3}\ m^3$

$2 \times 10^6 \leqslant$ number of bottles $\leqslant 2 \times 10^7$

liter	m^3
1	10^{-3}
x	2×10^3

$\Rightarrow x = \frac{2 \times 10^3}{10^{-3}} = 2 \times 10^6$

c) $\rho = 1000\ kg/m^3 = 10^3\ kg/m^3$

$\rho = \frac{m}{V'} \Rightarrow m = \rho V' \rightarrow m_{min} = 10^3 \times 2 \times 10^3 \Rightarrow m_{min} = 2 \times 10^6\ kg$

$m_{max} = 10^3 \times 2 \times 10^4 \Rightarrow m_{max} = 2 \times 10^7\ kg$

27 M Iron has a density of 7.87 g/cm^3, and the mass of an iron atom is 9.27×10^{-26} kg. If the atoms are spherical and tightly packed, (a) what is the volume of an iron atom and (b) what is the distance between the centers of adjacent atoms?

$\rho = 7{,}87\ g/cm^3$

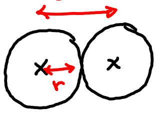
$m = 9{,}27 \times 10^{-26}\ \underset{10^3 g}{kg} = 9{,}27 \times 10^{-23}\ g$

a) $\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$

$V = \frac{9{,}27 \times 10^{-23}}{7{,}87} \simeq 1{,}18 \times 10^{-23}\ cm^3 = 1{,}18 \times 10^{-29}\ m^3$

$d = 2r$

b)



$V = \frac{4}{3}\pi r^3 \rightarrow r = \left(\frac{3}{4}\frac{V}{\pi}\right)^{1/3}$

$\rightarrow r = \sqrt[3]{\frac{3}{4} \times \frac{1{,}18 \times 10^{-29}}{3{,}14}} = \sqrt[3]{\underbrace{0{,}28}_{2{,}8 \times 10^{-1}} \times 10^{-29}} = \sqrt[3]{2{,}8 \times 10^{-30}} \simeq 1{,}41 \times 10^{-10}\ m \Rightarrow d \simeq 2{,}82 \times 10^{-10}\ m$

28 M A mole of atoms is 6.02×10^{23} atoms. To the nearest order of magnitude, how many moles of atoms are in a large domestic cat? The masses of a hydrogen atom, an oxygen atom, and a carbon atom are 1.0 u, 16 u, and 12 u, respectively. (*Hint:* Cats are sometimes known to kill a mole.)

$1 \text{ mol} = 6{,}02 \times 10^{23}$

$m = 10 \ \underbrace{kg}_{10^3 g} = 10^4 \ g$

$H_2O \ , \ C_6H_{12}O_6$

$\longrightarrow \dfrac{2\overbrace{m_H}^{2} + \overbrace{m_O}^{16} + \overbrace{m_C}^{12}}{4} = 7{,}25 \ u$

$m_H = 1 \ u$

$m_O = 16 \ u$

$m_C = 12 \ u$

Appendix D : $1u = 1{,}66 \times 10^{-24} \ g$

u	g
1	$1{,}66 \times 10^{-24}$
x	10^4

$\Rightarrow m_{Cat} = \dfrac{10^4}{1{,}66 \times 10^{-24}} = 6{,}0 \times 10^{27} \ u$

$\Rightarrow$ number of atoms : $N = \dfrac{6{,}0 \times 10^{27}}{7{,}25} \simeq 0{,}8 \times 10^{27}$

mol	atom
1	$6{,}02 \times 10^{23}$
x	$0{,}8 \times 10^{27}$

$\Rightarrow x = \dfrac{0{,}8 \times 10^{\cancel{27}\,4}}{6{,}02 \times \cancel{10^{23}}} \simeq \underbrace{0{,}1 \times 10^4}_{10^3} \text{ mol} \simeq \boxed{1 \text{ Kmol}}$

29 M On a spending spree in Malaysia, you buy an ox with a weight of 28.9 piculs in the local unit of weights: 1 picul = 100 gins, 1 gin = 16 tahils, 1 tahil = 10 chees, and 1 chee = 10 hoons. The weight of 1 hoon corresponds to a mass of 0.3779 g. When you arrange to ship the ox home to your astonished family, how much mass in kilograms must you declare on the shipping manifest? (*Hint:* Set up multiple chain-link conversions.)

$m = 28{,}9$ picul

1 picul = 100 gin

1 gin = 16 tahil

1 tahil = 10 chee

1 chee = 10 hoon

1 hoon = $0{,}3779$ g

$m = 28{,}9 \ \underbrace{\text{picul}}_{100 \ \underbrace{\text{gin}}_{16 \ \underbrace{\text{tahil}}_{10 \ \underbrace{\text{chee}}_{10 \ \underbrace{\text{hoon}}_{0{,}3779}}}}}$

$\Rightarrow m = 28{,}9 \times 100 \times 16 \times 10 \times 10 \times 0{,}3779 \simeq \underbrace{174{,}7}_{1{,}75 \times 10^2} \times 10^4 \ g = 1{,}75 \times 10^6 \ \underbrace{g}_{10^{-3} kg} = \boxed{1{,}75 \times 10^3 \ kg}$

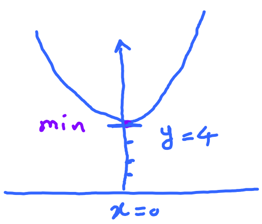
30 M CALC GO Water is poured into a container that has a small leak. The mass m of the water is given as a function of time t by $m = 5.00t^{0.8} - 3.00t + 20.00$, with $t \geq 0$, m in grams, and t in seconds. (a) At what time is the water mass greatest, and (b) what is that greatest mass? In kilograms per minute, what is the rate of mass change at (c) $t = 2.00$ s and (d) $t = 5.00$ s?

$m = 5t^{0{,}8} - 3t + 20 \qquad t \geq 0 \ , \ t : s \ , \ m : g$

Example : $y = x^2 + 4 \longrightarrow y' = \dfrac{dy}{dx} = 2x \longrightarrow y' = 0 \Rightarrow 2x = 0 \Rightarrow x = 0 \rightarrow$ min or max

$y'' = \dfrac{d^2y}{dx^2} = 2 > 0 \longrightarrow x = 0 : \text{min}$

$y|_{x=0} = 4$



a) $m' = \dfrac{dm}{dt} = 5 \times 0{,}8 \, t^{0{,}8-1} - 3 = 4t^{-0{,}2} - 3$

$dm/dt = 0 \Rightarrow 4t^{-0{,}2} = 3 \rightarrow \dfrac{4}{t^{0{,}2 = 1/5}} = 3 \Rightarrow t^{1/5} = 4/3 \Rightarrow (t^{1/5})^5 = (4/3)^5$

$\rightarrow \boxed{t \simeq 4,21 \text{ s}}$

$$m'' = \frac{d^2m}{dt^2} = 4 \times (-0,2)\, t^{-0,2-1} = -0,8\, t^{-1,2} = \frac{-0,8}{t^{1,2}} < 0 \rightarrow t = 4,21 \text{ s} \leftarrow \text{max}$$

b) $$m\Big|_{t=4,21} = 5,00 \times (4,21)^{0,8} - 3,00 \times 4,21 + 20 \simeq 23,2 \text{ g} \Rightarrow \boxed{m_{max} = 23,2 \text{ g}}$$

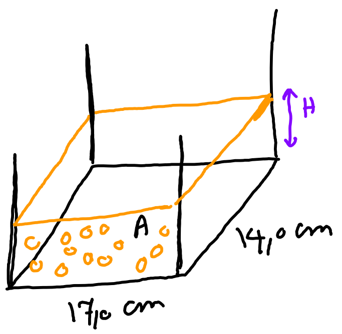
c) $$R = \frac{dm}{dt} = 4t^{-0,2} - 3$$

$$t = 2,00 \text{ s} \Rightarrow R\Big|_{t=2,00} = 4(2,00)^{-0,2} - 3 = 0,48 \text{ g/s} = 0,48 \frac{10^{-3}\text{ kg}}{\frac{1}{60}\text{ min}}$$

$$\Rightarrow R\Big|_{t=2,00} = 0,48 \times 60 \times 10^{-3} = \boxed{2,88 \times 10^{-2} \text{ kg/min}}$$

d) $$t = 5,00 \rightarrow R\Big|_{t=5,00} = 4(5,00)^{-0,2} - 3 = -0,101 \text{ g/s} = -0,101 \times 60 \times 10^{-3} = \boxed{-6,06 \times 10^{-3} \frac{\text{kg}}{\text{min}}}$$

31 H CALC A vertical container with base area measuring 14.0 cm by 17.0 cm is being filled with identical pieces of candy, each with a volume of 50.0 mm^3 and a mass of 0.0200 g. Assume that the volume of the empty spaces between the candies is negligible. If the height of the candies in the container increases at the rate of 0.250 cm/s, at what rate (kilograms per minute) does the mass of the candies in the container increase?



$v = 50,0 \text{ mm}^3$ $\quad R = \frac{dH}{dt} = 0,250 \text{ cm/s}$; $\frac{dM}{dt} = ?$

$m = 0,0200 \text{ g}$

$0,250$ cm/s

$$V = A \cdot H = 17,0 \times 14,0 \times H \Rightarrow \frac{dV}{dt} = 17,0 \times 14,0 \times \frac{dH}{dt}$$

$$\Rightarrow \frac{dV}{dt} \simeq 59,5 \text{ cm}^3/\text{s}$$

$$\rho_{candy} = \frac{m}{v} = \frac{0,0200 \text{ g}}{(50,0) \times 10^{-3} \text{ cm}^3} = \frac{2 \times \cancel{10^{-2}}}{5 \times \cancel{10^{-2}}} = 0,4 \text{ g/cm}^3$$

$$\rho = \frac{M}{V} \rightarrow M = \rho V \Rightarrow \frac{dM}{dt} = \rho \frac{dV}{dt} = 0,4 \times 59,5 \simeq 23,8 \text{ g/s}$$

$$\Rightarrow \frac{dM}{dt} = 23,8 \frac{10^{-3}\text{ kg}}{\frac{1}{60}\text{ min}} = \underbrace{23,8 \times 60}_{1,428} \times 10^{-3} \frac{\text{kg}}{\text{min}} \rightarrow \boxed{\frac{dM}{dt} \simeq 1,43 \times 10^{-3} \frac{\text{kg}}{\text{min}}}$$