10 E Until 1883, every city and town in the United States kept its own local time. Today, travelers reset their watches only when the time change equals 1.0 h. How far, on the average, must you travel in degrees of longitude between the time-zone boundaries at which your watch must be reset by 1.0 h? (*Hint:* Earth rotates 360° in about 24 h.)

$$\begin{array}{cc} 360^\circ & 24\,h \\ x & 1 \end{array} \longrightarrow x = \frac{360}{24} = \boxed{15^\circ}$$

11 E For about 10 years after the French Revolution, the French government attempted to base measures of time on multiples of ten: One week consisted of 10 days, one day consisted of 10 hours, one hour consisted of 100 minutes, and one minute consisted of 100 seconds. What are the ratios of (a) the French decimal week to the standard week and (b) the French decimal second to the standard second?

1 week = 10 day

1 day = 10 h

1 h = 100 min

1 min = 100 s

1 standard day = 1 decimal day = 1 day

$$a)\ \frac{\text{decimal week}}{\text{standard week}} = \frac{10\ \cancel{\text{day}}}{7\ \cancel{\text{day}}} = \frac{10}{7} \simeq 1{,}43$$

b) 1 standard day $= 24 \times 60 \times 60 = 86400$ standard second

1 decimal day $= 10 \times 10^2 \times 10^2 = 10^5$ decimal second

$$\Rightarrow \frac{1\ \cancel{\text{day}}}{1\ \cancel{\text{day}}} = \frac{86400}{10^5}\ \frac{\text{standard second}}{\text{decimal second}} \Rightarrow \frac{\text{decimal second}}{\text{standard second}} = \frac{86400}{10^5} = 0{,}864$$

12 E The fastest growing plant on record is a *Hesperoyucca whipplei* that grew 3.7 m in 14 days. What was its growth rate in micrometers per second?

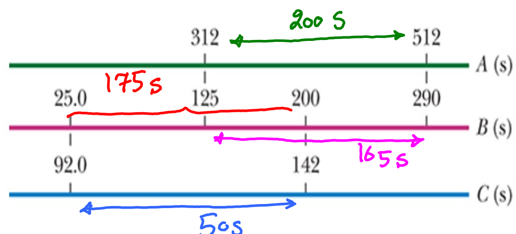
$t = 14$ day

$h = 3{,}7$ m

$$\Rightarrow R = \frac{3{,}7}{14}\ \frac{m}{day} \longrightarrow ?\ \frac{\mu m}{s}$$

$$\Rightarrow R = \frac{3{,}7}{14}\ \frac{m}{day} = \frac{3{,}7}{14} \times \frac{10^6\ \mu m}{\underbrace{24 \times 3600}_{86400}\ s} \simeq 3{,}1\ \mu m/s$$

13 E GO Three digital clocks *A*, *B*, and *C* run at different rates and do not have simultaneous readings of zero. Figure 1.3 shows simultaneous readings on pairs of the clocks for four occasions. (At the earliest occasion, for example, *B* reads 25.0 s and *C* reads 92.0 s.) If two events are 600 s apart on clock *A*, how far apart are they on (a) clock *B* and (b) clock *C*? (c) When clock *A* reads 400 s, what does clock *B* read? (d) When clock *C* reads 15.0 s, what does clock *B* read? (Assume negative readings for prezero times.)

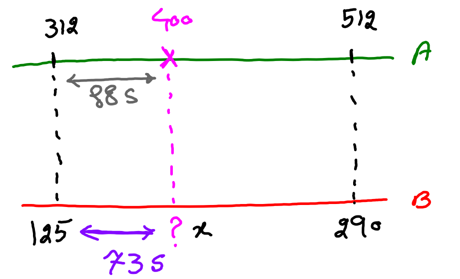


a) $\Delta t_A = 200$ s $\xrightarrow{\times 3}$ $\Delta t'_A = 600$ s

$\Delta t_B = 165$ s $\xrightarrow{\times 3}$ $\Delta t'_B = 165 \times 3 = 495$ s

b) $DT_B = 175\ s \quad , \quad DT'_B = 495$ $\quad \times 495/175$

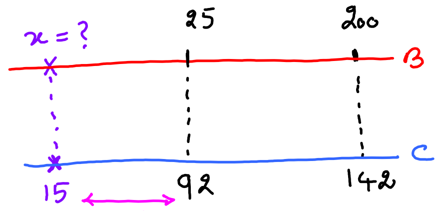
$DT_C = 50\ s \longrightarrow DT'_C = 50 \times \frac{495}{175} \simeq 141{,}43 = \boxed{141\ s}$



c) $Dt_A = 200\ s \quad , \quad Dt''_A = 88$ $\quad \times 88/200$

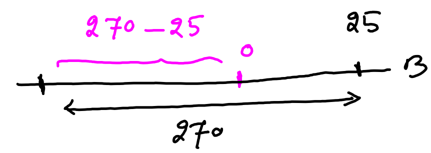
$Dt_B = 165\ s \longrightarrow Dt''_B = 165 \times \frac{88}{200} = 72{,}6\ s$

$\longrightarrow x = 125 + 73 = \boxed{198\ s}$



d) $DT_B = 175\ s \longrightarrow DT''_B = 175 \times \frac{77}{50} \simeq 269{,}5 = 270$

$DT_C = 50\ S \quad , \quad DT''_C = 77$ $\quad \times 77/50$



$\Longrightarrow x = -(270 - 25) = \boxed{-245\ s}$

14 E A lecture period (50 min) is close to 1 microcentury. (a) How long is a microcentury in minutes? (b) Using

$$\text{percentage difference} = \left(\frac{\text{actual} - \text{approximation}}{\text{actual}}\right)100,$$

find the percentage difference from the approximation.

a) $1\ \mu\text{century} = 10^{-6}\ \text{century} = 10^{-6} \times 100\ \text{year} = 10^{-4}\,(365{,}25 \times 24 \times 60) = 52{,}596 \simeq \boxed{52{,}6\ \text{min}}$

b) $\text{difference}\% = \frac{52{,}6 - 50}{52{,}6} \times 100 \simeq \boxed{4{,}9\%}$

15 E A fortnight is a charming English measure of time equal to 2.0 weeks (the word is a contraction of "fourteen nights"). That is a nice amount of time in pleasant company but perhaps a painful string of microseconds in unpleasant company. How many microseconds are in a fortnight?

$\text{fortnight} = 2\ \text{weeks} = 2 \times 7\ \text{day} = 14 \times 24 \times 60 \times 60\ s = \underbrace{1'209'600}_{1{,}209600 \times 10^{6}}\ s \rightarrow 10^{6}\ \mu s$

$\Longrightarrow 1\ \text{fortnigh} = \boxed{1{,}2 \times 10^{12}\ \mu s}$

16 E Time standards are now based on atomic clocks. A promising second standard is based on *pulsars*, which are rotating neutron stars (highly compact stars consisting only of neutrons). Some rotate at a rate that is highly stable, sending out a radio beacon that sweeps briefly across Earth once with each rotation, like a lighthouse beacon. Pulsar PSR 1937 + 21 is an example; it rotates once every 1.557 806 448 872 75 ± 3 ms, where the trailing ±3 indicates the uncertainty in the last decimal place (it does *not* mean ±3 ms). (a) How many rotations does PSR 1937 + 21 make in 7.00 days? (b) How much time does the pulsar take to rotate exactly one million times and (c) what is the associated uncertainty?

$t = 1{,}557806\cdots\ ms \longrightarrow 1\ s = 10^{3}\ ms$

a) $T = 7\ \text{days} = 7 \times 24 \times 3600\ s = 604800 \times 10^{3}\ ms$

$\rightarrow$ number of rotation $= \dfrac{604800 \times 10^3}{1{,}557806\ldots} \simeq 388238218 = 3{,}88 \times 10^8$

b) $t' = 1{,}557806\ldots \times \underbrace{10^6}_{\text{one million time}} \overset{10^{-3}\,s}{ms} = 1{,}557806\ldots \times 10^3 \; s$

c) $t = 1{,}557806448872 75 \mp 3 \times 10^{-14} \; ms$

$\Delta t = 3 \times 10^{-14} \; ms \times 10^6 = 3 \times 10^{-8} \; ms = 3 \times 10^{-11} \; s$

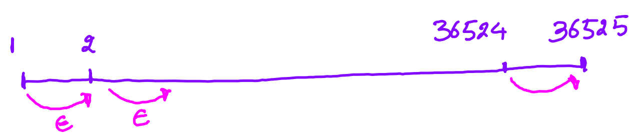
17 E SSM Five clocks are being tested in a laboratory. Exactly at noon, as determined by the WWV time signal, on successive days of a week the clocks read as in the following table. Rank the five clocks according to their relative value as good timekeepers, best to worst. Justify your choice.

Clock	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
A	12:36:40	12:36:56	12:37:12	12:37:27	12:37:44	12:37:59	12:38:14
B	11:59:59	12:00:02	11:59:57	12:00:07	12:00:02	11:59:56	12:00:03
C	15:50:45	15:51:43	15:52:41	15:53:39	15:54:37	15:55:35	15:56:33
D	12:03:59	12:02:52	12:01:45	12:00:38	11:59:31	11:58:24	11:57:17
E	12:03:59	12:02:49	12:01:54	12:01:52	12:01:32	12:01:22	12:01:12

	Mon – Sun	Tues – Mon	Wed – Tues	Thurs – Wed	Fri – Thurs	Sat – Fri	
A:	+16	+16	+15	+17	+15	+15	3
B:	+3	−5	+10	−5	−6	+7	4
C:	+58	+58	+58	+58	+58	+58	1
D:	−67	−67	−67	−67	−67	−67	2
E:	−70	−55	−2	−20	−10	−10	5

18 M Because Earth's rotation is gradually slowing, the length of each day increases: The day at the end of 1.0 century is 1.0 ms longer than the day at the start of the century. In 20 centuries, what is the total of the daily increases in time?

At the end of 1st century : $24h + 10^{-3}$ s $\rightarrow$



" " " 2nd " : $24h + 2 \times 10^{-3}$ s

⋮

20th " : $24h + 20 \times 10^{-3}$ s

number of days in a year $= 365{,}25$

" " " " century $=$

$365{,}25 \times 100 = 36525$

1st century, 1st day : $24h + 0$

2nd " : $24h + 1\epsilon$

3rd " : $24h + 2\epsilon$ $\rightarrow$ 20 century

⋮

36525th " : $24h + 10^{-3}$ s $\rightarrow (36525 - 1)\epsilon = 36524\,\epsilon \rightarrow \epsilon = \dfrac{10^{-3}}{36524}$ s

Long solution

1st century : $\sum$ daily increase $= \epsilon \xrightarrow{\epsilon} 2\epsilon \xrightarrow{\epsilon} 3\epsilon + \cdots + 36524\epsilon$

$$S_n = \frac{n(a_1 + a_n)}{2} \quad ; n = 36524 \;,\; a_1 = \epsilon \;,\; a_n = 36524\epsilon$$

$$\Rightarrow \sum \text{daily increase} = \frac{36524\,(\cancel{\epsilon} + \overset{10^{-3}}{36524\epsilon})}{2} \simeq 18{,}26 \text{ s}$$

2nd century : $\sum$ daily increase $= (10^{-3} + \epsilon) + (10^{-3} + 2\epsilon) + \cdots + (10^{-3} + \overbrace{36524\epsilon}^{10^{-3}})$

$$= 10^{-3} \times 36524 + \{\underbrace{\epsilon + 2\epsilon + 3\epsilon + \cdots + 36524\epsilon}_{18{,}26}\}$$

3rd century : $\sum$ daily increase $= 2 \times 10^{-3} \times 36524 + 18{,}26$

$\vdots$

20th century : $\sum$ daily increase $= 19 \times 10^{-3} \times 36524 + 18{,}26$

$$\text{total daily increase} = \overbrace{\{18{,}26\}}^{1st} + \overbrace{\{18{,}26 + 36524 \times 10^{-3}\}}^{2nd} + \overbrace{\{18{,}26 + 2 \times 10^{-3} \times 36524\}}^{3rd} + \cdots + \underbrace{\{18{,}26 + 19 \times 10^{-3} \times 36524\}}_{20th}$$

$$= 18{,}26 \times 20 + 36524 \times 10^{-3} \{\underbrace{1 + 2 + \cdots + 19}_{\frac{19 \times (1+19)}{2} = 190}\}$$

$$\Rightarrow \text{total daily increase} = 7304{,}95 \text{ s} \simeq \boxed{7305 \text{ s}}$$

Short solution

1st day — last day $= 36525 \times 20$

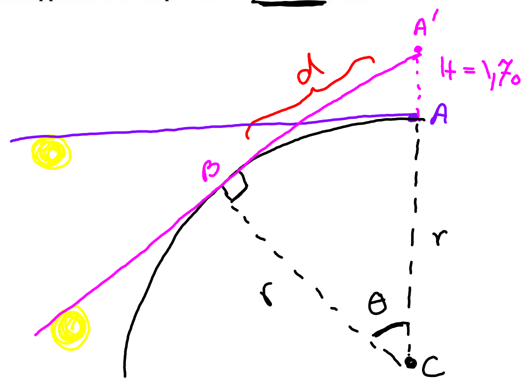
$\Delta t = 0$ — $\Delta t = 20 \times 10^{-3} = 0{,}02$ s

1st century — 20th century

$$\text{average} = \frac{0 + 0{,}02}{2} = 0{,}01$$

$$\text{total daily increase} = 36525 \times 20 \times 0{,}01 = \boxed{7305} \text{ s}$$

19 H Suppose that, while lying on a beach near the equator watching the Sun set over a calm ocean, you start a stopwatch just as the top of the Sun disappears. You then stand, elevating your eyes by a height $H = 1.70$ m, and stop the watch when the top of the Sun again disappears. If the elapsed time is $t = 11.1$ s, what is the radius r of Earth?



$\triangle A'BC: \quad (r+H)^2 = r^2 + d^2$

$H = 1,70$ m

$\cancel{r^2} + 2rH + \cancel{H^2} = \cancel{r^2} + d^2$

$H \ll r \longrightarrow \quad 2rH = d^2 \quad (1)$

$\triangle A'BC: \quad \tan\theta = \dfrac{d}{r} \Rightarrow d = r \cdot \tan\theta \quad (2)$

$(1), (2) \longrightarrow \quad 2\cancel{r}H = (r \cdot \tan\theta)^2 = r^{\cancel{2}} \tan^2\theta \Rightarrow 2H = r\tan^2\theta \Rightarrow r = \dfrac{2H}{\underbrace{\tan^2\theta}_{?}} \quad *$

$360^\circ \qquad 24h \rightarrow 24 \times 3600 = 86400$ s

$\theta \qquad 11,1 \Rightarrow \theta = \dfrac{11,1 \times 360^\circ}{86400} \simeq 0,047^\circ$

$*: \quad r = \dfrac{2 \times 1,70}{\tan^2(0,047^\circ)} \simeq 5,22 \times 10^6$ m