47 SSM An astronomical unit (AU) is the average distance between Earth and the Sun, approximately 1.50×10^8 km. The speed of light is about 3.0×10^8 m/s. Express the speed of light in astronomical units per minute.

$1\ Au = 1{,}50 \times 10^8\ \text{Km}\ (10^3\ m) = 1{,}50 \times 10^{11}\ m \longrightarrow 1\ m = \dfrac{1}{1{,}5 \times 10^{11}}\ Au$

$v_c = 3{,}0 \times 10^8\ m/s = ?\ Au/min \qquad 1\ min = 60\ s \longrightarrow 1\ s = \dfrac{1}{60}\ min$

$$v_c = 3{,}0 \times 10^8 \times \overset{m}{\frac{Au}{1{,}5 \times 10^{11}}} \times \frac{1}{\frac{1}{60}\ min}{}_{s} = \cancel{3{,}0}^{\,2} \times \cancel{10^8} \times \frac{60}{\cancel{1{,}5} \times \cancel{10^{11}}_{10^3}}\ Au/min = 120 \times 10^{-3} = \boxed{0{,}12\ Au/min}$$

$v_c = 0{,}12\ Au/min$

min	Au
1	0,12
x	1

$\longrightarrow x = \dfrac{1}{0{,}12} \simeq 8\ min$

distance between earth and sun

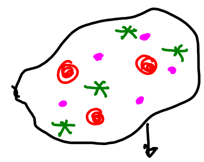
48 The common Eastern mole, a mammal, typically has a mass of 75 g, which corresponds to about 7.5 moles of atoms. (A mole of atoms is 6.02×10^{23} atoms.) In atomic mass units (u), what is the average mass of the atoms in the common Eastern mole?

$M = 75\ g \longrightarrow 7{,}5$ mole

$m \qquad 1$

$\longrightarrow m = \dfrac{75}{7{,}5} = 10\ g \longrightarrow$ mass of 1 mole

$1\ \text{mole} = 6{,}02 \times 10^{23}$ atom



average mass = ?

$$\text{average mass} = \frac{\cancel{10}}{6{,}02 \times \cancel{10^{23}}_{10^{22}}} \simeq \underbrace{0{,}166}_{1{,}66 \times 10^{-1}} \times 10^{-22}$$

$\longrightarrow$ average mass $= 1{,}66 \times 10^{-23}\ g$

Appendix D: $1\ u = 1{,}66 \times 10^{-24}\ g$

$\longrightarrow \boxed{\text{average mass} = 10\ u}$

49 A traditional unit of length in Japan is the ken (1 ken = 1.97 m). What are the ratios of (a) square kens to square meters and (b) cubic kens to cubic meters? What is the volume of a cylindrical water tank of height 5.50 kens and radius 3.00 kens in (c) cubic kens and (d) cubic meters?

$1\ \text{ken} = 1{,}97\ m$

a) $\dfrac{ken^2}{m^2} = \dfrac{(1{,}97\ m)^2}{m^2} = (1{,}97)^2 \simeq \boxed{3{,}88}$

b) $\dfrac{ken^3}{m^3} = \dfrac{(1{,}97\ m)^3}{m^3} = (1{,}97)^3 \simeq \boxed{7{,}65}$

c) $r = 3{,}00$ ken $\qquad V = \pi r^2 \cdot h = 3{,}14 \times (3{,}00)^2 \times 5{,}50 = 155{,}43 \simeq \boxed{1{,}55 \times 10^2\ ken^3}$

$h = 5{,}50$ ken

d) part b: $ken^3 = 7{,}65\ m^3 \longrightarrow V = 1{,}55 \times 10^2\ \underbrace{ken^3}_{7{,}65\ m^3} = 1{,}55 \times 7{,}65 \times 10^2\ m^3$

$\longrightarrow V = 1189{,}04 = \boxed{1{,}19 \times 10^3\ m^3}$

50 You receive orders to sail due east for 24.5 mi to put your salvage ship directly over a sunken pirate ship. However, when your divers probe the ocean floor at that location and find no evidence of a ship, you radio back to your source of information, only to discover that the sailing distance was supposed to be 24.5 *nautical miles*, not regular miles. Use the Length table in Appendix D to calculate how far horizontally you are from the pirate ship in kilometers.

$d = 24{,}5 \text{ mi}$

$\xleftrightarrow{24{,}5 \times 1{,}609 \text{ km}} \xleftrightarrow{d'}$

$\xleftrightarrow{24{,}5 \times 1{,}852 \text{ km}}$: real

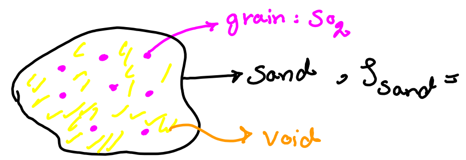
Appendix D: $1 \text{ nautical mile} = 1{,}852 \text{ km}$

$1 \text{ mile} = 1{,}609 \text{ km}$

$$d' = 24{,}5 \times 1{,}852 - 24{,}5 \times 1{,}609 = 24{,}5 \times \underbrace{(1{,}852 - 1{,}609)}_{0{,}243} \Rightarrow \boxed{d' \simeq 5{,}95 \text{ km}}$$

51 *Density and liquefaction.* A heavy object can sink into the ground during an earthquake if the shaking causes the ground to undergo *liquefaction*, in which the soil grains experience little friction as they slide over one another. The ground is then effectively quicksand. The possibility of liquefaction in sandy ground can be predicted in terms of the *void ratio* e for a sample of the ground: $e = V_{voids}/V_{grains}$. Here, V_{grains} is the total volume of the sand grains in the sample and V_{voids} is the total volume between the grains (in the *voids*). If e exceeds a critical value of 0.80, liquefaction can occur during an earthquake. What is the corresponding sand density ρ_{sand}? Solid silicon dioxide (the primary component of sand) has a density of $\rho_{SiO_2} = 2.600 \times 10^3 \text{ kg/m}^3$.

$$e = \frac{V_{void}}{V_{grain}} \Rightarrow V_{void} = e\, V_{grain}$$



$$V_{sand} = V_{grain} + V_{void} = V_{grain} + e\,V_{grain} = (1+e)\, V_{grain} \quad (3)$$

$$\rho_{sand} = \frac{m_{sand} = m_{grain} + m_{void}}{V_{sand}} \quad (2) \qquad m_{void} \to 0$$

$$\rho_{SO_2} = \frac{m_{SO_2} = m_{grain}}{V_{SO_2} = V_{grain}} \rightarrow m_{grain} = \rho_{SO_2} \cdot V_{grain} \quad (1)$$

$$(1), (2), (3) \rightarrow \rho_{sand} = \frac{\rho_{SO_2} \cdot \cancel{V_{grain}}}{(1+e) \cdot \cancel{V_{grain}}} = \frac{\rho_{SO_2}}{(1+e)}$$

$\rho_{SO_2} = 2{,}600 \times 10^3 \text{ kg/m}^3$

$e = 0{,}80$

$$\Rightarrow \rho_{sand} = \frac{2{,}600 \times 10^3}{1{,}80} \simeq \boxed{1{,}44 \times 10^3 \text{ kg/m}^3}$$

52 *Billion and trillion.* Until 1974, the U.S. and the U.K. used the same names to mean different large numbers. Here are two examples: In American English a billion means a number with 9 zeros after the 1 and in British English it formerly meant a number with 12 zeros after the 1. In American English a trillion means a number with 12 zeros after the 1 and in British English it formerly meant a number with 18 zeros after the 1. In scientific notation with the prefixes in Table 1.1.2, what is 4.0 billion meters in (a) the American use and (b) the former British use? What is 5.0 trillion meters in (c) the American use and (d) the former British use?

American English : 1×10^9 ← one billion → 1×10^{12} ← one trillion

British " : 1×10^{12} ← one billion → 1×10^{18} ← one trillion

Table 1.1.2 : $10^9 \longrightarrow$ giga (G)

$10^{12} \longrightarrow$ tera (T)

$10^{18} \longrightarrow$ exa (E)

a) U.S. : $4{,}0 \times 10^9 \text{ m} = \boxed{4{,}0 \text{ Gm}}$

b) U.K. : $4{,}0 \times 10^{12} \text{ m} = \boxed{4{,}0 \text{ Tm}}$

c) U.S. : $5{,}0 \times 10^{12} \text{ m} = \boxed{5{,}0 \text{ Tm}}$

d) U.K. : $5{,}0 \times 10^{18} \text{ m} = \boxed{5{,}0 \text{ Em}}$

53 *Townships.* In the United States, real estate can be measured in terms of *townships*: 1 township = 36 mi², 1 mi² = 640 acres, 1 acre = 4840 yd², 1 yd² = 9 ft². If you own 3.0 townships, how many square feet of real estate do you own?

$$\left.\begin{array}{l} 1 \text{ township} = 36 \text{ mi}^2 \\ 1 \text{ mi}^2 = 640 \text{ acre} \\ 1 \text{ acre} = 4840 \text{ yd}^2 \\ 1 \text{ yd}^2 = 9 \text{ ft}^2 \end{array}\right\} \rightarrow 1 \text{ township} = 36 \times 360 \times 4840 \times 9 = 100'362'240 \text{ ft}^2$$

$A = 3$ township $\simeq 3{,}01 \times 10^8$ ft²

54 *Measures of a man.* Leonardo da Vinci, renowned for his understanding of human anatomy, valued the measures of a man stated by Vitruvius Pollio, a Roman architect and engineer of the first century BC: four fingers make one palm, four palms make one foot, six palms make one cubit, and four cubits make a man's height. If we take a finger width to be 0.75 in., what then are (a) the length of a man's foot and (b) the height of a man, both in centimeters?

4 finger = 1 palm

4 palm = 1 foot *

6 palm = 1 cubit

4 cubit = 1 height

$\underset{\downarrow}{w}$ = ,175 in
finger

a) $1 \text{ foot} = 4 \underbrace{\text{palm}}_{4 \text{ finger}} = 16 \text{ finger} = 16 \times {,}75 = 12 \text{ in}$ → 1 foot = 12 x 2,54 = 30,48 ≃ 35 cm

Appendix D: 1 in = 2,54 cm

b) $1 \text{ height} = 4 \underbrace{\text{cubit}}_{6 \text{ palm}} = 24 \text{ palm} = 24 \times 1/4 \text{ foot} = 6 \underbrace{\text{foot}}_{} \simeq 183 \text{ cm}$

*: 1 palm = 1/4 foot

part a: 30,48 cm

55 *Dog years.* Dog owners like to convert the age of a dog (dubbed *dog years*) to the usual meaning of years to account for the more rapid aging of dogs. One measure of the aging process in both dogs and humans is the rate at which the DNA changes in a process called methylation. Research on that process shows that after the first year, the equivalent age of a dog is given by

$$\text{equivalent age} = 16 \ln(\text{dog years}) + 31,$$

where ln is the natural logarithm. What then is the equivalent age of a dog on its 13th birthday?

dog year = 13 → equivalent age = $16 \underbrace{\ln(13)}_{\simeq 2{,}56} + 31 = 16 \times 2{,}56 + 31 = 71{,}03 \simeq 71$ years

56 *Galactic years.* The time the Solar System takes to circle around the center of the Milky Way galaxy, a galactic year, is about 230 My. In galactic years, how long ago did (a) the *Tyrannosaurus rex* dinosaurs live (67 My ago), (b) the first major ice age occur (2.2 Gy ago), and (c) Earth form (4.54 Gy ago)?

1 galactic year = 230×10^6 year

a) $t = 67 \times 10^6 \text{ year} = \dfrac{67 \times \cancel{10^6}}{230 \times \cancel{10^6}} \simeq {,}129$ galactic years

b) $t' = 2{,}2 \times 10^9 \text{ year} = \dfrac{2{,}2 \times \cancel{10^9}\ 10^3}{230 \times \cancel{10^6}} \simeq 9{,}6$ " "

c) $t'' = 4{,}54 \times 10^9 \text{ year} = \dfrac{4{,}54 \times \cancel{10^9}\ 10^3}{230 \times \cancel{10^6}} \simeq 19{,}7$ " "

57 *Planck time.* The smallest time interval defined in physics is the Planck time $t_P = 5.39 \times 10^{-44}$ s, which is the time required for light to travel across a certain length in a vacuum. The universe began with the big bang 13.772 billion years ago. What is the number of Planck times since that beginning?

$t_P = 5{,}39 \times 10^{-44}$ s

$T = 13{,}772 \times 10^{9}$ year $= 13{,}772 \times 10^{9} \times 365{,}25 \times 24 \times 3600 \simeq 4{,}34 \times 10^{8} \times 10^{9} = 4{,}34 \times 10^{17}$ s

$$\boxed{T/t_P} = \frac{4{,}34 \times 10^{17}}{5{,}39 \times 10^{-44}} \simeq \underbrace{0{,}805}_{8{,}05 \times 10^{-1}} \times 10^{17} \times 10^{44} = \boxed{8{,}05 \times 10^{60}}$$

58 *20,000 Leagues Under the Sea.* In Jules Verne's classic science fiction story (published as a serial from 1869 to 1870), Captain Nemo travels in his underwater ship *Nautilus* through the seas of the world for a distance of 20,000 leagues, where a (metric) league is equal to 4.000 km. Assume Earth is spherical with a radius of 6378 km. How many times could Nemo have traveled around Earth?

$d = 20'000$ league

1 league $= 4000$ km

$\rightarrow d = 20'000 \times 4000 = 8 \times 10^{7}$ km

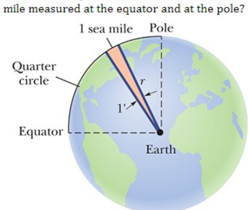
$r = 6378$ km $\longrightarrow C = 2\pi r = 2 \times 3{,}14 \times 6378 \simeq 40'053$ km

$$\rightarrow \boxed{n =} \frac{8 \times 10^{7}}{40'053} \simeq \boxed{1997 \text{ times}}$$

59 *Sea mile.* A sea mile is a commonly used measure of distance in navigation but, unlike the *nautical mile*, it does not have a fixed value because it depends on the latitude at which it is measured. It is the distance measured along any given longitude that subtends 1 arc minute, as measured from Earth's center (Fig. 1.5). That distance depends on the radius r of Earth at that point, but because Earth is not a perfect sphere but is wider at the equator and has slightly flattened polar regions, the radius depends on the latitude. At the equator, the radius is 6378 km; at the pole it is 6356 km. What is the difference in a sea mile measured at the equator and at the pole?

$r_{eq} = 6378$ km

$r_{pole} = 6356$ km

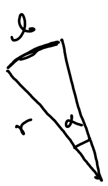


$\alpha = 1$ arc min $= ?$ rad

$1° = 60$ arc min $= \frac{\pi}{180}$ rad

$180°$ — π rad

$1°$ — $x = \frac{\pi}{180}$ rad



$l = r\alpha$, $\alpha \rightarrow$ rad

$$1 \text{ arc min} = \frac{3{,}14}{180 \times 60} \simeq 2{,}9 \times 10^{-4} \text{ rad} \longleftarrow 1 \text{ arc min} = \frac{\pi/180}{60}$$

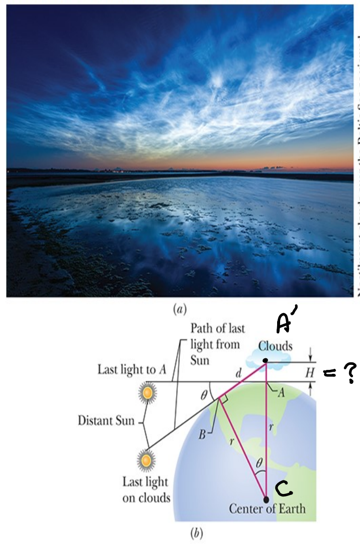
equator : $l = 6378 \times 2{,}9 \times 10^{-4}$

pole : $l' = 6356 \times 2{,}9 \times 10^{-4}$

$$\rightarrow \Delta l = l - l' = 2{,}9 \times 10^{-4} \times \underbrace{(6378 - 6356)}_{22 \text{ km}}$$

$$\rightarrow \boxed{\Delta l =} \underbrace{63{,}8}_{6{,}38 \times 10} \times 10^{-4} \text{ km} = 6{,}38 \times \underbrace{10^{-3} \text{ km}}_{m} = \boxed{6{,}38 \text{ m}}$$

60 *Noctilucent clouds.* Soon after the huge 1883 volcanic explosion of Krakatoa Island (near Java in the southeast Pacific), silvery, blue clouds began to appear nightly in the Northern Hemisphere early at night. The explosion was so violent that it hurled dust to the *mesosphere*, a cool portion of the atmosphere located well above the stratosphere. There water collected and froze on the dust to form the particles that made the first of these clouds. Termed *noctilucent clouds* ("night shining"), these clouds are now appearing frequently (Fig. 1.6*a*), signaling a major change in Earth's atmosphere, not because of volcanic explosions, but because of the increased production of methane by industries, rice paddies, landfills, and livestock flatulence.



Noctilucent clouds over the Baltic Sea as viewed from Laboe, Germany, 2019. Source: Matthias Süßen. Licensed under CC BY-SA 4.0

$r = 6378$ km

$t = 38$ min

Figure 1.6 Problem 60. (*a*) Noctilucent clouds. (*b*) Sunlight reaching the observer and the clouds.

The clouds are visible after sunset because they are in the upper portion of the atmosphere that is still illuminated by sunlight. Figure 1.6*b* shows the situation for an observer at point *A* who sees the clouds overhead 38 min after sunset. The two lines of light are tangent to Earth's surface at *A* and *B*, at radius *r* from Earth's center. Earth rotates through angle *θ* between the two lines of light. What is the height *H* of the clouds?

D

$A'BC: \ (r+H)^2 = d^2 + r^2$

$\cancel{r^2} + 2rH + H^2 = \underbrace{d^2}_{(r\tan\theta)^2} + \cancel{r^2}$

$\tan\theta = \dfrac{d}{r} \Rightarrow d = r.\tan\theta$

θ → rotation angle

min

24×60 → $360°$

38 → $\theta = \dfrac{360 \times 38}{24 \times 60} = 9{,}5°$

$2rH + H^2 = r^2\tan^2\theta$ $\quad H = ?$

$\underset{a=1}{H^2} + \underset{b}{2r}H \underbrace{- r^2\tan^2\theta}_{c} = 0$

$ax^2 + 2bx + c = 0 \longrightarrow x = \dfrac{-b \mp \sqrt{b^2 - ac}}{a}$

$H = -r \mp \sqrt{r^2 + r^2\tan^2\theta} = -r \mp \sqrt{r^2(1+\tan^2\theta)} = -r \mp r\sqrt{\underbrace{1 + \dfrac{\sin^2\theta}{\cos^2\theta}}} = -r \mp r\sqrt{\dfrac{1}{\cos^2\theta}}$

$\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$\Rightarrow H = -r \mp \dfrac{r}{\cos\theta}$

$\rightarrow \boxed{H} = r\left(-1 + \dfrac{1}{\cos\theta}\right) = 6378 \times \left(-1 + \dfrac{1}{\cos(9{,}5°)}\right) \quad \cos(9{,}5°) \simeq 0{,}986 \quad = 6378(-1 + 1{,}014) \simeq \boxed{90 \text{ km}}$

61 *Class time, the long of it.* For a common four-year undergraduate program, what are the total number of (a) hours and (b) seconds spent in class? Enter your answer in scientific notation.

$t = 4$ year

a) $t = 4 \times 365{,}25 \times 24 \simeq \boxed{3{,}5 \times 10^4 \text{ h}}$

b) $t = 4 \times 365{,}25 \times 24 \times 3600 \simeq \boxed{1{,}3 \times 10^8 \text{ s}}$